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RESEARCH ARTICLE

Topology-Diversified Planning-Stage Siting of Grid-Forming Inverters With Post-Event Current-Proxy Feasibility Checks

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ABSTRACT Grid-forming (GFM) inverters can improve the response of inverter-dominated distribution feeders, but their benefit depends strongly on feeder topology, disturbance location, local voltage support, and converter current capability. This paper proposes a planning-stage GFM siting framework that combines vulnerability-based screening with topology diversification and feasibility checks. Candidate placements are evaluated using a reduced-order phasor-domain planning model with an aggregated frequency-response equation and positive-sequence power-flow checks. The selected placement minimizes a robust base/low-inertia maximum dynamic-security-index objective ($DSI_{\max}$) over the selection event set and is subsequently checked using a voltage-adjusted current-capability proxy. The framework is evaluated using disjoint hold-out events, feasible-set comparisons, sensitivity checks, and current-limited power-flow feasibility checks. On the IEEE 33-bus feeder, the proposed placement improves the composite dynamic security index (DSI_{comp}) relative to a topology-only distributed baseline by 4.3%/5.0% on the selection set and 8.6%/8.8% on the hold-out set under base/low-inertia cases. On the high-GFM-penetration simplified IEEE 123-node-derived reconstruction, the corresponding improvements are 0.9%/0.9% on the selection set and 22.0%/22.1% on the five-event hold-out set evaluated in this reduced planning case. For the post-event power-flow snapshots evaluated in this study, the selected placements yield feasible current-limited power-flow solutions under the declared 1.2 pu voltage-adjusted current-proxy constraint, with maximum recorded proxy values of 0.9244 pu and 0.9172 pu in the two test systems. The results support the method as a deterministic planning-stage screening tool with retained verification records; detailed unbalanced electromagnetic, protection, and manufacturer-specific controller validation remains outside the planning-stage model.

INDEX TERMS Grid-forming inverter, inverter placement, distribution feeder, current capability, dynamic security index, radial feeder, topology-aware screening.

NOMENCLATURE

CVI Critical vulnerability index.
DER Distributed energy resource.
DSI Dynamic security index.
EMT Electromagnetic transient.

GFL Grid-following.
GFM Grid-forming.
LOO Leave-one-out.
PLL Phase-locked loop.
RoCoF Rate of change of frequency.
SCR Short-circuit ratio.

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- M1 Grid-following-only case (no GFM).
 M2 Topology-only distributed baseline.
 M3 Proposed CVI-screened robust $DSI_{\max}$ placement.

Sets and Indices

- $\mathcal{G}, \mathcal{B}, \mathcal{E}$ Feeder graph, bus set, and branch set.
 $\mathcal{B}_{\text{cand}}$ Screened candidate bus set.
 $\mathcal{C}_{\text{sel}}, \mathcal{C}_{\text{test}}$ Selection and hold-out event sets.
 $\mathcal{L}$ Inertia-case set {base, low}.
 $\Pi(d_{\min})$ Distance-feasible placement set.
 π, π^* A placement and the selected placement.
 i Event index.
 b, b_i Bus index and event bus.

Parameters and Constants

- N_g Number of GFM units.
 S_b GFM unit rating.
 $S_{\text{base}}, S_{\text{ref}}$ System MVA base and SCR reference MVA.
 $d_{\min}$ Minimum topological (hop) distance.
 τ Topological influence radius.
 $H_{\text{eq}}, H_{\text{gfm}}$ Upstream-equivalent and GFM inertia constants.
 D_{load}, K_p Load damping and GFM active-power damping coefficient.
 $I_{\max}$ Voltage-adjusted current-proxy limit.
 K Screened candidate-set size.
 λ Positive-sequence load-scaling factor.
 α Mean/worst weighting in DSI_{comp} .
 $\beta_1, \beta_2, \beta_3$ CVI feature weights.
 w_f, w_r, w_v, w_s DSI component weights.
 T_w RoCoF evaluation window.

Variables and Metrics

- $\Delta\omega_i, \Delta P_i$ Frequency deviation and active-power imbalance.
 $H_{\text{eff},i}, D_{\text{eff},i}$ Effective inertia and damping for event i .
 $d(b_m, b_n)$ Graph-hop distance between two buses.
 DSI_i Event-level dynamic security index.
 $DSI_{\max}$ Worst-event DSI over an event set.
 DSI_{comp} Composite (mean/worst) DSI.
 $DSI_{\text{comp}}^{\text{pool}}$ Pooled base/low-inertia composite DSI.
 $|\Delta f|_{\max,i}$ Peak absolute frequency deviation.
 $|\text{RoCoF}|_{\max,i}$ Peak absolute rate of change of frequency.
 $|\Delta V|_{\max,i}^{\text{post}}$ Peak post-event voltage deviation.
 $t_{\text{settle},i}, f_{\text{ss},i}$ Settling time and steady-state frequency.
 CVI_b Critical vulnerability index at bus b .
 SCR_b Short-circuit ratio at bus b .
 $P_b, Q_b, |V_b|$ Active power, reactive power, and voltage magnitude at bus b .
 $I_{\text{pu},b}$ Voltage-adjusted current proxy at bus b .

- P_b^+, Q_b^+ Positive-sequence retained active and reactive load.
 $|\Pi|$ Number of distance-feasible placements.

I. INTRODUCTION

Distribution networks are increasingly expected to host inverter-based generation, storage, and controllable loads while maintaining acceptable voltage and frequency response under weak-grid conditions [1]. Grid-forming (GFM) inverters are a promising technology for this operating regime because they can provide voltage-source behavior, fast active-power support, and local voltage regulation [1], [2]. However, their performance is not determined only by installed capacity. In a radial distribution feeder, the same GFM rating can produce materially different response quality depending on where it is connected, which buses are disturbed, and how much feeder impedance lies between the GFM and the affected region.

This creates a placement problem. Exhaustive evaluation of all multi-GFM placements is possible for very small feeders, but it becomes unattractive as the candidate set grows and when each placement must be tested across many disturbances and operating conditions. A purely electrical vulnerability score can reduce the search space, yet high-vulnerability buses may cluster in one branch of a compact radial feeder. A placement that looks strong under one event set may then generalize poorly to disturbances in other parts of the feeder. Conversely, imposing too much spatial diversification on a large feeder may discard strong candidates without improving dynamic response. A practical siting method therefore needs more than a single optimal placement: it should explain how candidates were screened, whether the rule was fixed before the final comparison, how the result transfers to events that were not used for selection, and whether current-capability feasibility is respected.

The objective of this paper is a planning-stage version of the GFM siting problem. Given a fixed total GFM capacity and a fixed number of identical units, the goal is to choose GFM buses that improve the worst event-level dynamic response on a selection event set while retaining performance on a disjoint hold-out event set. The study deliberately avoids claiming exact controller-level validation. The dynamic layer is a reduced-order phasor-domain planning model, the simplified IEEE 123-node-derived reconstruction is a positive-sequence reconstruction from tabular topology and load data, and the current check is a feasibility check based on reactive-power-limited power flow. These modeling choices support deterministic planning studies with retained verification records, while detailed unbalanced electromagnetic transient analysis remains outside the present planning-stage model.

This paper makes four contributions.

- It proposes a deterministic CVI-to-reduced-order-evaluation siting workflow that reports both the global

CVI rank and the final screening rank of selected GFM buses.

- It identifies a topology-diversification trade-off in radial distribution-feeder GFM siting: in the compact feeder, depth-stratified CVI screening with a larger minimum hop distance avoids downstream-cluster overfitting, whereas in the larger feeder, pure CVI screening with a smaller distance constraint preserves the useful high-CVI candidate set. This trade-off is demonstrated through screening-policy ablation.
- It evaluates placements using $DSI_{\max}$ on the selection set and reports DSI_{comp} on both selection and hold-out sets, avoiding a result that is supported only by average-case behavior.
- It reports hold-out tests, feasible-set comparisons, screening-policy ablations, distance-sensitivity checks, current-capability feasibility checks, and equation-level consistency checks of the reported composite index.

The rest of the paper is organized as follows. Section II summarizes related work and the gap addressed here. Section III defines the planning problem and validation protocol. Section IV presents the reduced-order response model, current-capability check, CVI, and dynamic response metric. Section V describes the proposed framework. Section VI reports the test systems and study settings. Section VII presents the numerical results. Section VIII discusses engineering interpretation, verification records, and limitations, and Section IX concludes the paper.

II. RELATED WORK AND RESEARCH GAP

GFM control has been studied from several perspectives, including virtual synchronous machines, droop-controlled voltage-source converters, synchronverters, dispatchable virtual oscillator control, and matching-control formulations [3], [4], [5], [6]. These studies establish that converter control can emulate some stabilizing characteristics of synchronous generation, but the distribution-feeder placement problem remains distinct. The planning engineer does not only ask whether a GFM controller is stable at a given bus. The engineer must decide where limited GFM capacity should be installed so that a diverse set of feeder disturbances is handled with acceptable voltage, frequency, and current behavior.

Existing inverter and distributed-energy-resource (DER) placement studies often use voltage sensitivity, short-circuit strength, power-loss reduction, hosting capacity, or composite vulnerability indices [7], [8], [9], [10]. Recent studies have also begun to address GFM placement and sizing directly [11], [12]. Reference [11] focuses on optimal allocation for voltage-stability improvement in a rural weak-grid setting, whereas the present work emphasizes radial distribution-feeder candidate screening with declared hop-distance diversification and disjoint hold-out validation. Reference [12] addresses multi-criteria placement and sizing of utility-scale GFM converters, while this study fixes the GFM rating and isolates the siting effect through

a topology-aware planning screen and current-proxy feasibility check. A related grid-strength-based placement study uses a generalized short-circuit-ratio criterion and Laplacian eigenvalue analysis for transmission-level phase-locked-loop (PLL)-integrated systems [13]; the present work instead combines an SCR-based screening feature with topology-diversified candidate enumeration tailored to radial distribution feeders. Static screening metrics are valuable because they are inexpensive to compute and provide physical intuition. However, a static score cannot by itself certify dynamic response. A high-CVI bus may be electrically weak, but three top-ranked buses may lie close to each other in a radial feeder, leaving other zones unsupported. A method that simply selects the top three scores can therefore be fragile, especially on compact feeders where a downstream branch dominates the vulnerability ranking.

Dynamic-security-based placement methods address part of this issue by simulating candidate placements under disturbances. The challenge is that dynamic evaluation is more expensive and can be sensitive to the event set used for selection. If the same events are used for both tuning and headline reporting, the result can overstate generality. In addition, placement studies can become difficult to interpret if a feeder-specific rule is introduced only after observing unfavorable results. A defensible method should distinguish pre-specified policy choices from post-hoc diagnostics.

Current capability is another important gap. GFM inverters are power-electronic devices with current limits, and their voltage-support benefit depends on the active and reactive operating point [14]. Interconnection standards such as IEEE Std 1547 define requirements for DER interfaces and grid-support behavior, but they do not by themselves determine where limited GFM resources should be placed in a radial feeder [15]. A placement that improves a voltage index by implicitly requiring excessive reactive injection is not practically meaningful. Detailed current-limiter dynamics are manufacturer- and controller-dependent, but a planning-stage paper should at least verify that the selected operating points remain within a declared current-capability proxy under the evaluated events.

The framework in this paper is designed around these gaps. It keeps the CVI screening layer because it is physically interpretable and computationally efficient. It adds topology diversification only through a pre-specified two-feeder case-study policy. It uses reduced-order planning evaluation to select among feasible multi-GFM placements. It reports hold-out performance and negative ablation results rather than only the strongest table entries. Finally, it includes a current-proxy feasibility check and states exactly what that check does and does not prove.

III. PROBLEM FORMULATION

A. NETWORK, EVENTS, AND CANDIDATE PLACEMENTS

Consider a radial distribution feeder represented by a graph $\mathcal{G} = (\mathcal{B}, \mathcal{E})$, where $\mathcal{B}$ is the set of buses and $\mathcal{E}$ is the

set of energized branches and closed bus-bus switches. The substation bus is excluded from the GFM siting candidates. A placement is denoted by

$$\pi = \{b_1, b_2, \dots, b_{N_g}\}, \quad b_j \in \mathcal{B}_{\text{cand}}. \quad (1)$$

where $N_g = 3$ in this study and $\mathcal{B}_{\text{cand}}$ is the screened candidate set. All GFM units have the same MVA rating, so this paper studies the siting effect under fixed total GFM capacity rather than simultaneous siting and sizing.

Events are divided into two disjoint sets. The selection set $\mathcal{C}_{\text{sel}}$ is used to choose the placement. The hold-out set $\mathcal{C}_{\text{test}}$ is used only after the placement has been fixed. Events include line outages, load trips, and load-step disturbances. In radial networks, an outage may isolate downstream load; therefore, the retained contingency-definition records explicitly report disconnected load for each outage event.

B. OBJECTIVE AND FEASIBILITY

The feasible placement set is constrained by a minimum topological distance:

$$\begin{aligned} \Pi(d_{\min}) &= \{\pi \subseteq \mathcal{B}_{\text{cand}} : |\pi| = N_g, \\ &\quad d(b_m, b_n) \geq d_{\min}, \\ &\quad \forall b_m, b_n \in \pi, m < n\}. \end{aligned} \quad (2)$$

Here, $d(b_m, b_n)$ denotes the graph-hop distance between buses b_m and b_n on the active feeder topology. The graph distance used in the distance constraint is computed on the active energized graph after excluding open switches and inactive auxiliary records. In the final implementation, the selected placement minimizes the worst event-level dynamic security index over both base- and low-inertia design cases on the selection set:

$$\begin{aligned} \pi^* &= \arg \min_{\pi \in \Pi(d_{\min})} \max_{\ell \in \mathcal{L}} DSI_{\max}^{(\ell)}(\pi; \mathcal{C}_{\text{sel}}), \\ \mathcal{L} &= \{\text{base}, \text{low}\}. \end{aligned} \quad (3)$$

The selected placement is then evaluated on $\mathcal{C}_{\text{test}}$ without re-optimization.

The validation protocol has four requirements. First, the candidate policy must be specified before the final comparative evaluation. Second, the feasible-set comparison must use the same candidate set and obey the same distance constraint as the proposed method. Third, the distance-sensitivity study must re-optimize the placement at each $d_{\min}$ so that feasible-set monotonicity is meaningful. Fourth, the current-capability check must use the same GFM rating and active dispatch as the dynamic study.

C. PLANNING-STAGE INTERPRETATION

The problem in (3) is not framed as a complete operational-security problem. It is a siting screen that should be used before detailed controller studies. This distinction matters because a distribution utility may need to compare many candidate storage or inverter interconnection points before manufacturer-specific controls, protection settings,

or final dispatch schedules are known. A planning screen should therefore be computationally light, physically interpretable, and conservative enough to avoid clearly infeasible recommendations.

The proposed method follows that philosophy. The CVI layer narrows the search space using feeder quantities that are available from a standard power-flow model. The topology rule prevents the candidate list from becoming an unexamined cluster. The dynamic layer then tests only feasible three-bus combinations rather than every non-substation bus triplet. Finally, the validation layer checks whether the selected result is sensitive to event selection, feasible-set alternatives, and current capability. The outcome is not a certificate of field readiness; it is a ranked and internally verified placement recommendation that can be passed to a higher-fidelity study.

The method also separates design choices from diagnostic checks. The two-feeder case-study policy, candidate strategy, number of GFM units, GFM rating, current limit, and event sets are specified before the final comparative evaluation. Policy ablations and distance sweeps are reported afterward to explain the result. This ordering supports reproducibility and protocol consistency because it reduces the risk that a rule is silently tuned to one unfavorable benchmark outcome.

IV. MODELING AND METRICS

A. AGGREGATED FREQUENCY RESPONSE

The study uses a reduced-order phasor-domain planning model. The upstream system is represented by a weak-grid equivalent, and feeder events create an active-power imbalance that drives the frequency state. The aggregated swing equation follows standard frequency-response modeling practice [16], [17]. For event i ,

$$2H_{\text{eff},i} \frac{d\Delta\omega_i}{dt} = -\Delta P_i - D_{\text{eff},i} \Delta\omega_i, \quad (4)$$

Here, ΔP_i is expressed on the system MVA base. The effective inertia and damping include distance-weighted GFM support:

$$H_{\text{eff},i} = H_{\text{eq}} + H_{\text{gfm}} \sum_{b \in \pi} \frac{S_b}{S_{\text{base}}} \exp\left[-\frac{d(b_i, b)}{\tau}\right], \quad (5)$$

$$D_{\text{eff},i} = D_{\text{load}} + K_p \sum_{b \in \pi} \frac{S_b}{S_{\text{base}}} \exp\left[-\frac{d(b_i, b)}{\tau}\right]. \quad (6)$$

Here, b_i is the event bus, S_b is the GFM rating, and τ is the topological influence radius. The exponential hop-distance term is a planning proxy rather than a physical propagation model. It penalizes electrically remote GFM support in radial feeders while keeping the screening model topology-driven and deterministic. The topological influence radius $\tau = 12$ is used as a declared planning-model constant for the two case studies. The numerical conclusions should therefore be interpreted under this reduced-order distance-decay assumption. A local planning-stage τ sweep over 8 to 20 has been added to test the robustness of the selected placement;

TABLE 1. Reduced-order planning-model parameters and DSI normalization constants.

Symbol	Meaning	Value or unit
ΔP_i	Net active-power deficit, positive for load increase or generation loss	pu on 10 MVA base
$\Delta \omega_i$	Frequency deviation	pu, $f = f_0(1 + \Delta \omega)$
H_{eq}	Upstream equivalent inertia	4 s (base), 2 s (low-inertia)
H_{gfm}	GFM virtual-inertia coefficient	4 s (fixed)
D_{load}	Aggregate load damping coefficient	1.0 pu
K_p	Equivalent active-power damping coefficient for droop-like response	20.0 pu/pu
τ	Topological influence radius	12 hops
I_{max}	Planning current-proxy limit	1.2 pu
Δf_{lim}	Frequency-deviation normalization constant	0.5 Hz
$RoCoF_{lim}$	RoCoF normalization constant	1.0 Hz/s
ΔV_{lim}	Voltage-deviation normalization constant	0.05 pu
$t_{settle,lim}$	Settling-time normalization constant	4.0 s
w_f, w_r, w_v, w_s	DSI component weights	0.25 each
α	Mean-worst weighting factor in DSI_{comp}	0.5

it leaves the selected placement unchanged in both feeders, so τ is a robustness-checked planning constant rather than a fitted value, as reported in the robustness table. Calibration of τ against higher-fidelity EMT or measured feeder responses remains future work. The K_p coefficient in the reduced-order swing-equation model represents the equivalent active-power damping contribution associated with droop-like GFM response in this planning model; it is not a separate manufacturer-specific droop-controller equation.

The reduced-order swing equation and distance-weighted effective inertia together emulate the aggregate behavior of a droop-controlled GFM inverter with virtual-inertia augmentation, similar in spirit to virtual-synchronous-machine formulations [3], [5]. The model does not represent controller-specific dynamics such as inner-loop current control, virtual impedance, or PLL-free synchronization, which are deferred to electromagnetic-transient (EMT) validation (see Section VIII-C). The parameters in eqs. (4)–(6) are summarized in Table 1. The fixed study settings used for reproducibility are $S_{base} = 10$ MVA, $N_g = 3$, $S_b = 1.5$ MVA per GFM unit, $K = 10$, active dispatch equal to 70% of rating, $\lambda = 0.5$ for the IEEE 123-node-derived reduced-load case, an 8 s simulation horizon, a 0.5 s f_{ss} averaging window, a 0.05 Hz settling threshold, and a 0.01 MW CVI voltage-sensitivity perturbation.

B. VOLTAGE AND CURRENT-CAPABILITY FEASIBILITY

Voltage response is estimated using positive-sequence power flow with constant-PQ loads using pandapower [18]. GFM units are represented as voltage-controlled generators with reactive-power limits. The active dispatch is fixed at 70% of rating. This dispatch level represents a mid-loading operating point that retains 30% apparent-power headroom for reactive support and current-proxy slack; sensitivity to dispatch level is outside the scope of the present planning-stage case study. Given planning current-proxy limit $I_{max} = 1.2$ pu and local voltage magnitude $|V_b|$, the reactive capability is constrained by

$$|Q_b| \leq \sqrt{(S_b I_{max} |V_b|)^2 - P_b^2}. \quad (7)$$

If $|P_b| > S_b I_{max} |V_b|$, the operating point is marked infeasible before assigning reactive capability. The voltage-adjusted current proxy is then computed as

$$I_{pu,b} = \frac{\sqrt{P_b^2 + Q_b^2}}{S_b |V_b|}. \quad (8)$$

Equations (7) and (8) define a current-limited power-flow feasibility screen rather than a dynamic current-limiter validation. The solver enforces the reactive capability implied by the declared current-proxy limit. Therefore, the resulting current-proxy values should not be interpreted as evidence that the siting algorithm dynamically prevents overcurrent. Instead, they indicate whether the selected post-event operating points remain feasible under the imposed converter capability constraint. The current proxy in eqs. (7) and (8) constrains the post-event power-flow operating point but does not model the dynamic saturation of active-current or fault-current limiting, active/reactive current-priority allocation, thermal limits, or protection behavior during the frequency transient; these are deferred to EMT-level study. Therefore, the DSI results should be interpreted as reduced-order dynamic screening results subject to a separate steady-state current-capability feasibility check. Although actual transient periods involve controller-dependent current saturation and recovery algorithms such as those proposed in [14], the proposed steady-state proxy serves as a computationally efficient preliminary filter. It excludes placements that are infeasible even in the post-event power-flow condition before more expensive EMT-level multi-GFM validation is attempted.

C. DYNAMIC SECURITY INDEX

Each event produces four normalized response components:

$$DSI_i = w_f \frac{|\Delta f|_{max,i}}{\Delta f_{lim}} + w_r \frac{|RoCoF|_{max,i}}{RoCoF_{lim}} + w_v \frac{|\Delta V|_{max,i}^{post}}{\Delta V_{lim}} + w_s \frac{t_{settle,i}}{t_{settle,lim}}, \quad (9)$$

where $w_f = w_r = w_v = w_s = 0.25$, $\Delta f_{lim} = 0.5$ Hz, $RoCoF_{lim} = 1.0$ Hz/s, $\Delta V_{lim} = 0.05$ pu, and $t_{settle,lim} = 4$ s. Equal weights are used to avoid tuning the index toward one

response channel in this planning-stage comparison. These constants define a relative planning metric and are not IEEE Std 1547 ride-through, trip, or compliance thresholds [15].

$$|\Delta f|_{\max,i} = \max_{t \geq t_0} |f_i(t) - f_0|, \quad (10)$$

$$|RoCoF|_{\max,i} = \max_{t \geq t_0} \left| \frac{f_i(t + T_w) - f_i(t)}{T_w} \right|, \quad T_w = 0.1 \text{ s}, \quad (11)$$

$$|\Delta V|_{\max,i}^{\text{post}} = \max_{b \in \mathcal{B}} |V_{b,i}^{\text{post}}| - 1.0. \quad (12)$$

Here t_0 is the disturbance time. The voltage term is computed from the post-event positive-sequence power-flow snapshot and is referenced to nominal voltage. The DSI therefore combines time-domain frequency-response quantities with a quasi-steady post-event voltage-deviation quantity. This mixed dynamic/static construction is intentional for planning-stage screening but should not be interpreted as a voltage-transient model. Because the planning screen evaluates post-event voltage acceptability rather than only disturbance-induced voltage change, the voltage component is referenced to nominal voltage instead of the pre-event voltage profile. The final frequency $f_{ss,i}$ is estimated as the mean over the last 0.5 s of the simulation window. The settling time $t_{\text{settle},i}$ is the first time after the disturbance for which $|f_i(t) - f_{ss,i}| \leq 0.05$ Hz remains true through the rest of the 8 s simulation window; if this condition is not met, the value is censored at the simulation horizon.

The placement-level worst event is

$$DSI_{\max}(\pi; \mathcal{C}) = \max_{i \in \mathcal{C}} DSI_i(\pi). \quad (13)$$

The reported composite index is

$$DSI_{\text{comp}}(\pi; \mathcal{C}) = \alpha \frac{1}{|\mathcal{C}|} \sum_{i \in \mathcal{C}} DSI_i(\pi) + (1 - \alpha) DSI_{\max}(\pi; \mathcal{C}). \quad (14)$$

Here, $\alpha = 0.5$ in this study. The optimizer uses $DSI_{\max}$ because a planning screen should not hide a severe event behind favorable average behavior. DSI_{comp} is reported because it summarizes mean-worst response quality and is easier to compare across methods. Because DSI_i is a weighted composite metric, it is used for relative planning comparison rather than compliance screening. A low composite value does not by itself certify that every individual response channel satisfies an operational hard limit; detailed hard-limit checks and protection studies remain part of subsequent operational validation.

The pooled variant used only for the DSI-normalization sensitivity table aggregates base- and low-inertia event-level values into a single scalar:

$$DSI_{\text{comp}}^{\text{pool}}(\pi; \mathcal{C}_{\text{sel}}) = \alpha \frac{1}{|\mathcal{L}||\mathcal{C}_{\text{sel}}|} \sum_{\ell \in \mathcal{L}} \sum_{i \in \mathcal{C}_{\text{sel}}} DSI_i^{(\ell)}(\pi)$$

$$+ (1 - \alpha) \max_{\ell \in \mathcal{L}, i \in \mathcal{C}_{\text{sel}}} DSI_i^{(\ell)}(\pi), \quad (15)$$

where $\mathcal{L} = \{\text{base}, \text{low}\}$. This pooled metric is not used for placement selection and is not the base/low separated DSI_{comp} reported in the main performance table. The pooled metric is used only for the normalization-sensitivity diagnostic because the purpose of Table 7 is to test whether the relative M2–M3 ordering is preserved across the combined base/low-inertia selection-event set, not to replace the base/low-separated performance reporting in Table 4.

D. CRITICAL VULNERABILITY INDEX

Eligible buses are ranked by a critical vulnerability index:

$$CVI_b = \beta_1 \hat{x}_{1,b} + \beta_2 \hat{x}_{2,b} + \beta_3 \hat{x}_{3,b}. \quad (16)$$

The components are $x_{1,b} = 1/SCR_b$, $x_{2,b} = |\partial V_b / \partial P_b|$, and $x_{3,b}$ equal to local load concentration. The SCR is computed as $SCR_b = S_{sc,b}/S_{\text{ref}}$ with $S_{\text{ref}} = 1.5$ MVA, using the positive-sequence short-circuit calculation available in the power-flow model. The voltage-sensitivity term is computed by a finite active-power perturbation of 0.01 MW at each eligible bus followed by positive-sequence power flow, and the sensitivity is evaluated as $|\Delta V_b / \Delta P_b|$. The local load-concentration term is computed as the active load at bus b plus the active loads at its one-hop energized neighbors, divided by the total active load over eligible buses. Min-max normalization is performed over eligible buses; if the range of a feature is zero, the normalized value of that feature is set to zero for all eligible buses. The hat denotes this normalized value, and $(\beta_1, \beta_2, \beta_3) = (0.4, 0.3, 0.3)$. These weights give slightly larger emphasis to short-circuit weakness while retaining voltage sensitivity and load concentration as co-equal secondary features; the later perturbation diagnostic checks that the reported candidate set is not produced by a knife-edge weight choice. The SCR term is used as a relative screening feature under the selected reference MVA; it should not be interpreted as an absolute transmission-level short-circuit-strength statement.

V. PROPOSED FRAMEWORK

Fig. 1 summarizes the workflow. The first stage builds the network model and computes CVI features. The second stage forms the candidate set according to a pre-specified two-feeder case-study policy. In this study, the IEEE 33-bus feeder is treated as the compact case and the simplified IEEE 123-node-derived reconstruction as the larger case. The corresponding screening policies are fixed before the final comparative evaluation: the compact case uses depth-stratified CVI screening with $d_{\min} = 3$, and the larger case uses pure CVI screening with $d_{\min} = 2$. This policy is a documented case-study choice rather than a universal feeder-classification standard.

For the compact feeder, the depth-stratified candidate set is constructed deterministically. Eligible buses are first ranked by CVI, and the highest $\min(N_{\text{elig}}, 3K)$ buses form the stratification pool. The pool is divided into near, middle, and remote

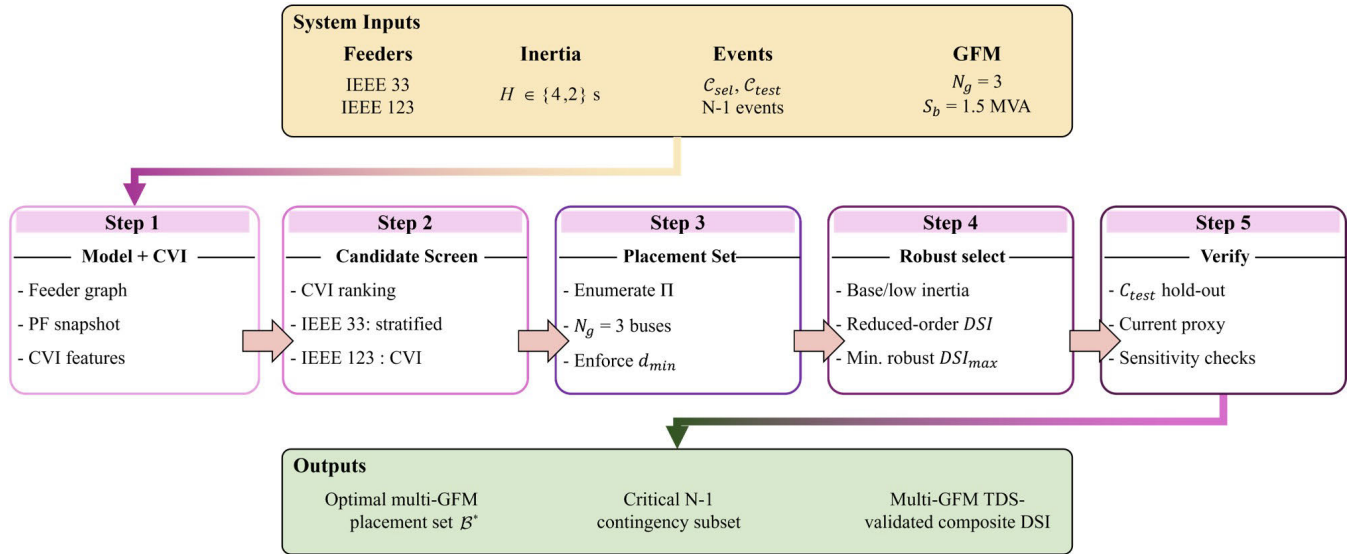


FIGURE 1. Proposed five-step planning-stage siting workflow. The inputs define the feeder cases, inertia scenarios, selection and hold-out event sets, and GFM settings. The workflow builds the planning model and CVI features, screens candidate buses, enumerates distance-feasible placements, selects the robust base/low-inertia $DSI_{\max}$ placement on the selection set, and retains hold-out, current-proxy, feasible-set, sensitivity, and ablation records for reporting.

groups using topological-depth tercile boundaries measured from the substation. For $K = 10$, the group quotas are 4, 3, and 3 buses from the near, middle, and remote groups, respectively, summing to $K = 10$. Within each group, buses are selected by descending CVI; any shortage caused by an empty group is filled by the remaining pool buses in global CVI order. The resulting candidate set is fixed before any DSI result is evaluated.

After candidate screening, all feasible three-bus placements satisfying (2) are enumerated. The reduced-order planning evaluation is then run for each feasible placement and each event in C_{sel} . The selected placement is the minimizer of (3). This exhaustive enumeration over the screened set is intentionally simple. It avoids the need to explain a metaheuristic and keeps the result internally verifiable: the retained verification records include the candidate list, feasible placement count, selected combination, and objective value.

The framework also generates diagnostic results that are not used to choose the final placement. A screening-policy ablation compares pure CVI and depth-stratified CVI under two distance constraints. The full feasible-set objective distribution is reported for the same candidate set and distance rule used by M3. A $d_{\min}$ sweep re-optimizes the placement after each change in the minimum hop distance. These diagnostics are included because they reveal whether the final rule is robust or merely a favorable special case.

A. COMPUTATIONAL PROCEDURE

The implementation is intentionally deterministic. After the candidate set is fixed, the feasible placements are generated by combinations of N_g buses and filtered by hop distance.

If the screened set has K buses, the unconstrained number of placements is $\binom{K}{N_g}$; in this study $K = 10$ and $N_g = 3$, so the upper bound is 120 placements before the distance filter. The screened candidate-set size $K = 10$ is chosen to balance two requirements: the finite enumeration must remain small enough for exact evaluation under all selection events, and the candidate pool must remain large enough to admit non-trivial topology diversification. For the IEEE 33-bus stratification rule, $K = 10$ yields at least three buses per depth group while keeping the search below 120 unconstrained triplets. The present study fixes $K = 10$ to keep the candidate envelope interpretable and exactly enumerable while retaining non-trivial depth diversity. Broader candidate-size sensitivity is outside the scope of the present two-feeder case study.

The resulting search space, at most 120 unconstrained triplets, is small enough that exact enumeration is preferable to a genetic algorithm, particle swarm method, or other stochastic optimizer. Because every feasible placement in the declared candidate envelope is evaluated directly, a stochastic metaheuristic would only approximate the same finite discrete search rather than define a stronger comparison. Exact enumeration also makes the result deterministic because the selected combination is not affected by random seeds or optimizer stopping tolerances.

For each feasible placement, the code evaluates all selection events under base and low inertia. The placement objective is the maximum $DSI_{\max}$ across these two inertia cases on C_{sel} , while the paper reports DSI_{comp} separately for selection and hold-out sets. The current-proxy check is run after the placement has been selected, using the same unit ratings and active-power dispatch. This sequencing avoids a

TABLE 2. Study systems, active graph records, event sets, and pre-specified screening policies.

System	Bus records	Active graph nodes	Edges used for hop distance	Open switches	Radiality check	Load MW	GFM MVA	Total GFM MVA	GFM/load	$ C_{sel} / C_{test} $	Screening policy	d_{min}
IEEE 33	33	33	32	0	$ \mathcal{E} = \mathcal{B} - 1$	3.715	3×1.5	4.5	1.21	12/4	stratified CVI	3
IEEE 123	128	125	124	5	$ \mathcal{E} = \mathcal{B} - 1$	1.745	3×1.5	4.5	2.58	15/5	CVI	2

circular procedure in which the current result influences the candidate policy after the fact.

The generated verification records are part of the workflow. They include the screened candidate table, feasible placement objective table, contingency table, validation gates, feasible-set objective distribution, distance-sensitivity sweep, policy-ablation summary, beta-perturbation diagnostic, capacity sensitivity, DSI-normalization sensitivity, and current-proxy summary. The manuscript compresses these records into a smaller set of main tables and figures; the underlying event definitions, candidate lists, objective tables, validation-gate records, and post-event current-proxy summaries are available from the corresponding author upon reasonable request.

VI. CASE STUDY SETUP

A. TEST FEEDERS AND GFM SETTINGS

The first test system is the standard balanced IEEE 33-bus radial feeder. The second is a simplified IEEE 123-node-derived reconstruction created from tabular line, load, and switch data from the IEEE distribution test-feeder family [19], [20]. For brevity, the standard IEEE 33-bus feeder and the simplified IEEE 123-node-derived reconstruction are denoted as IEEE 33 and IEEE 123, respectively, in tables, figures, and repeated result discussions. The abbreviation IEEE 123 in this paper refers only to the simplified positive-sequence reconstruction used in this study, not to an exact phase-domain IEEE 123-node benchmark implementation. The reconstruction preserves the radial topology and aggregate spot-load distribution used by the study, but it does not include exact phase-domain line-code matrices, voltage regulators, capacitor banks, transformer models, or load-voltage dependency. The reconstructed model contains 128 bus records because switch-endpoint records from the tabular data are retained during the positive-sequence conversion. The active graph has 125 energized nodes rather than 128 because the inactive switch-endpoint records corresponding to tabular nodes 251, 350, and 451 are not reachable from the substation after normally open switches are excluded. These inactive records are retained only for mapping consistency and are not intended to represent additional physical load buses in the active planning graph. Therefore, the case is described as a simplified IEEE 123-node-derived reconstruction rather than an exact IEEE 123-node benchmark implementation. Table 2 summarizes the feeder records, aggregate loads, GFM ratings, event-set sizes, and pre-specified screening policies used in the two case studies.

The 1.745 MW aggregate load reported for IEEE 123 is the reduced planning load used after phase aggregation and load prescaling. In the positive-sequence reconstruction, the retained load at bus b is obtained as

$$P_b^+ = \lambda \sum_{\phi \in \Phi_b^{\text{ret}}} P_{b,\phi}, \quad (17)$$

$$Q_b^+ = \lambda \sum_{\phi \in \Phi_b^{\text{ret}}} Q_{b,\phi}, \quad (18)$$

where Φ_b^{ret} denotes all phase spot-load records retained from the tabular load file for bus b , and $\lambda = 0.5$ is the load-scaling factor used in this reduced planning case. No nonzero spot-load record is removed by the phase aggregation step; the simplification is that all load models are represented as constant-PQ positive-sequence loads. The tabular phase-load sum is 3.490 MW and 1.920 Mvar; therefore, with $\lambda = 0.5$,

$$\sum_b P_b^+ = 1.745 \text{ MW}, \quad \sum_b Q_b^+ = 0.960 \text{ Mvar}.$$

Phase aggregation is used strictly to isolate the topological relationship between candidate GFM locations and feeder-wide response metrics in a planning-stage screen. Phase-domain unbalanced analysis, phase-balancing assessment, and EMT-level controller validation remain in subsequent operational design stages. Because this simplified model does not reproduce the full phase-domain IEEE 123-node benchmark load exactly, the case should be interpreted as a topology-preserving reduced planning case rather than as an exact loading replica of the IEEE 123-node feeder. Table 2 therefore reports both total GFM MVA and the GFM-MVA-to-load-MW ratio.

The IEEE 123 case is a high-GFM-penetration reduced planning case because the positive-sequence load scaling yields 1.745 MW of total load while the installed GFM rating is 4.5 MVA. The IEEE 123 results are therefore interpreted as a test of the siting workflow under a high converter-to-load ratio, not as a typical penetration level for the original phase-domain IEEE 123 benchmark. Consistent with the formulation, hop distances in Table 2 are computed on the active energized graph after excluding open switches and inactive switch-endpoint records.

Each study uses three identical GFM units rated at 1.5 MVA. The selected rating is used to compare siting policies under a fixed GFM capacity and should not be interpreted as an optimized penetration level. The comparison methods are: M1, a GFL-only case with no GFM units; M2, a deterministic topology-only distributed baseline;

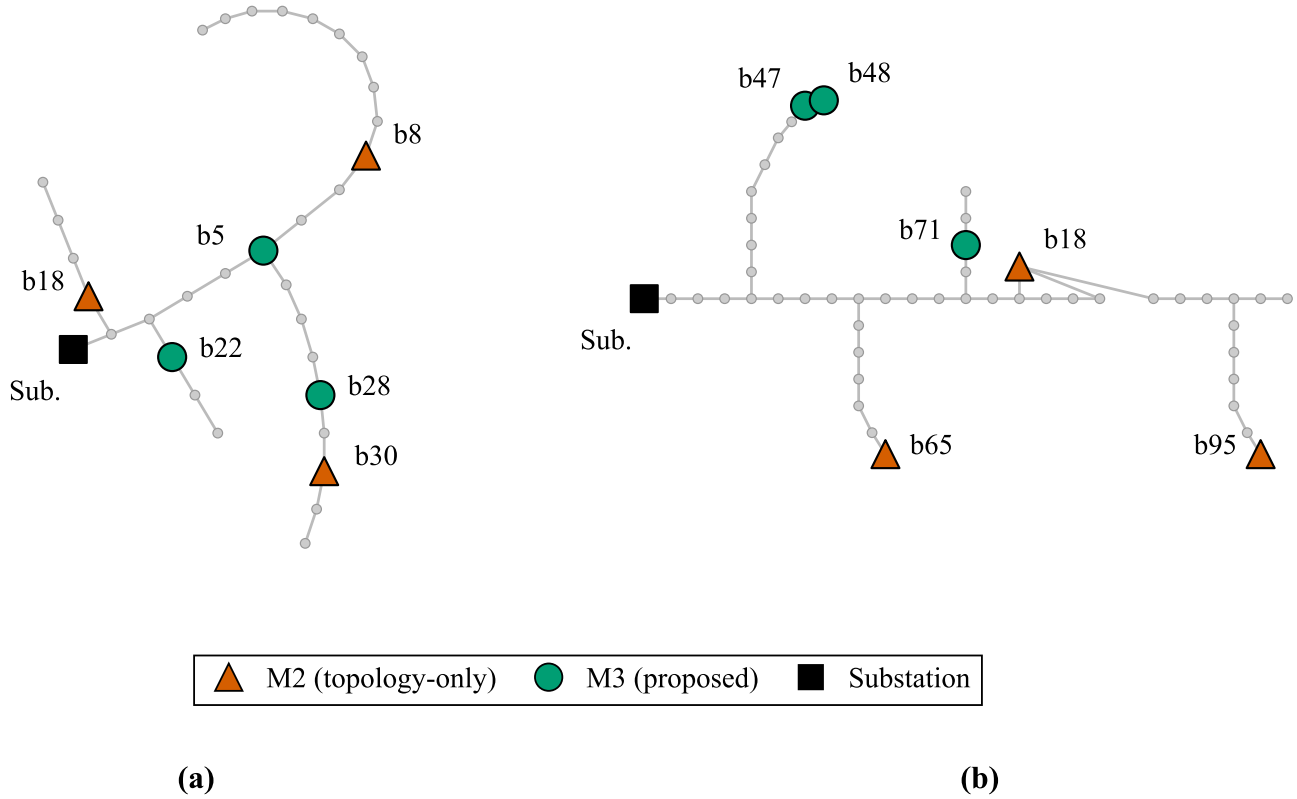


FIGURE 2. M2 and M3 locations on the evaluated IEEE 33 and IEEE 123 feeder graphs. M2 denotes the topology-only distributed baseline, and M3 denotes the proposed CVI-screened robust $DSI_{\max}$ placement. The labels identify the selected buses and the substation. The layout is used for planning interpretation rather than as a geographic map; (a) IEEE 33 and (b) IEEE 123.

TABLE 3. Placement definitions used in the main comparison. Depths denote the topological hop distance from the substation on the active feeder graph.

System	Method	GFM buses	Depths	Global CVI ranks	Screening ranks	Selection rule
IEEE 33	M2	b8, b18, b30	8, 2, 11	–	–	Topology-only distributed baseline
IEEE 33	M3	b5, b22, b28	5, 3, 9	22, 21, 13	10, 9, 7	CVI-screened robust $DSI_{\max}$ optimization
IEEE 33	CVI-top3	b17, b16, b15	17, 16, 15	1, 2, 3	1, 2, 3	Top-3 CVI without distance constraint
IEEE 123	M2	b18, b65, b95	5, 14, 20	–	–	Topology-only distributed baseline
IEEE 123	M3	b47, b48, b71	11, 11, 12	5, 8, 7	5, 8, 7	CVI-screened robust $DSI_{\max}$ optimization
IEEE 123	CVI-top3	b46, b85, b76	10, 14, 14	1, 2, 3	1, 2, 3	Top-3 CVI without distance constraint

M3, the CVI-screened robust $DSI_{\max}$ optimization result; and an unconstrained CVI-top3 diagnostic that selects the three highest-CVI buses without the final topology policy. Base and low-inertia cases are evaluated with $H_{\text{eq}} = 4$ s and $H_{\text{eq}} = 2$ s, respectively. The low-inertia case halves the upstream equivalent inertia to represent reduced synchronous-machine contribution in a sensitivity study, following standard aggregate frequency-response practice [16], [17] rather than claiming a specific measured feeder condition. Only H_{eq} is varied between the base- and low-inertia cases; H_{gfm} is kept fixed to isolate the effect of upstream inertia reduction.

B. EVENT-SET CONSTRUCTION

The selection and hold-out event sets are intentionally disjoint. The selection set contains the disturbances used to

optimize the placement, while the hold-out set contains disturbances that are not evaluated until the placement is fixed. This split is not a replacement for a large probabilistic scenario study, but it is a minimum requirement for a credible placement paper. Without a hold-out set, a placement can appear strong simply because it was tuned to a narrow set of events.

The event types are chosen to exercise different response channels. Load-step events primarily test active-power and voltage support near the disturbed bus. Load-trip events test the response to sudden power imbalance and the ability of the reduced-order frequency model to represent the aggregate effect. Line-outage events test topological sensitivity and, in radial feeders, may disconnect downstream load. The code records the disconnected load so that an outage is not misinterpreted as a purely local branch-status change.

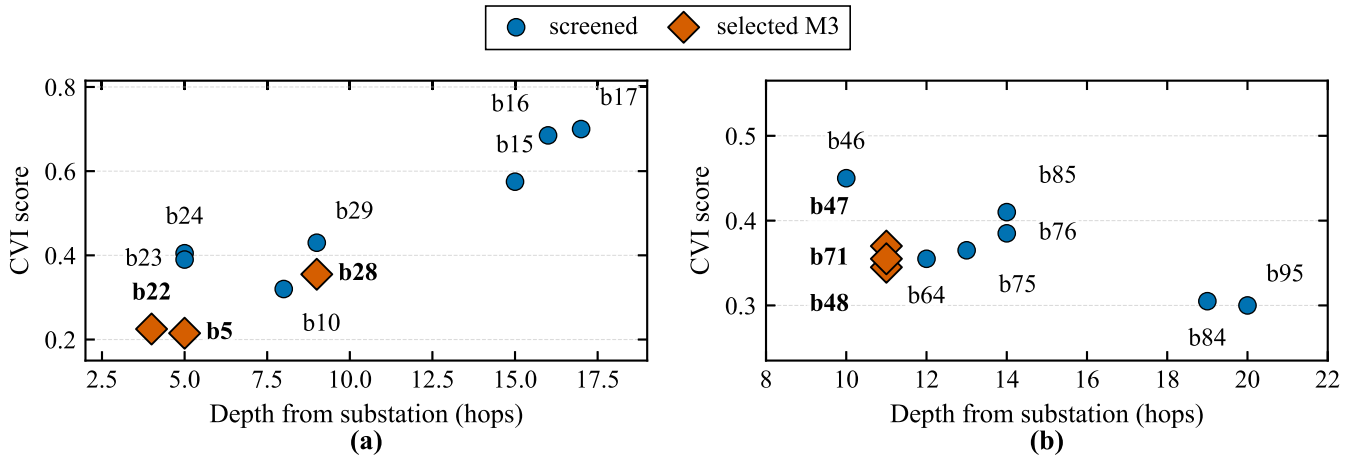


FIGURE 3. CVI score versus topological depth for screened candidates in IEEE 33 and IEEE 123. Selected M3 buses are highlighted. IEEE 33 uses depth-stratified screening to avoid a compact downstream cluster, whereas IEEE 123 uses pure CVI screening under the pre-specified two-feeder case-study policy; (a) IEEE 33 and (b) IEEE 123.

The base and low-inertia cases are evaluated with the same event definitions. This matters because a placement should not be credited with robustness if the low-inertia case is tested against a different or easier event set. The paper therefore reports base and low-inertia columns side by side for both C_{sel} and C_{test} . The complete contingency definitions, including event ID, event type, location, active/reactive disturbance magnitude, disconnected load for line outages, and selection/hold-out assignment, are retained as event-definition records and are available from the corresponding author upon reasonable request.

C. CANDIDATE LOCATIONS

Fig. 2 shows the M2 and M3 locations on the two feeder graphs, and Table 3 lists the exact buses and selection rules. Bus labels in the tables and figures use the internal zero-based pandapower indices with the substation denoted by b0. Thus, IEEE 33 b22 corresponds to conventional one-based bus 23 and has a three-hop path from the substation through b1 and b2 in the evaluated graph. The selected M3 locations and the distributed M2 baseline buses are graphically mapped onto the feeder topologies in Fig. 2 to illustrate how the proposed placement changes the spatial coverage relative to the topology-only distributed baseline. M2 is generated by a deterministic topology-only rule recorded in the study configuration. Each feeder is partitioned by electrical/topological distance from the substation into three representative feeder sections, and one representative load bus is selected from the central or terminal portion of each section to provide separated feeder-region coverage. When more than one bus could serve as the same regional representative under this topology-only rule, the retained study configuration resolves the ambiguity by the fixed representative bus declared in Table 3; no performance-based tie-breaking is applied. The selected M2 triplets are fixed before any comparative evaluation and are declared in Table 3. No CVI, DSI, event-response,

or current-proxy information is used. Therefore, M2 serves as a reproducible distributed baseline rather than a tuned performance baseline, and rank entries are omitted for M2 because it is not selected from the CVI-screened candidate list. The CVI-top3 diagnostic uses the three highest-CVI buses without imposing the final distance constraint. The IEEE 33 result places the M3 units at buses 5, 22, and 28. This is not the pure top-CVI cluster; it is a diversified placement produced by depth-stratified screening and $d_{min} = 3$. The IEEE 123 result places the M3 units at buses 47, 48, and 71. Although buses 47 and 48 are adjacent in the bus numbering, the graph distance policy is applied to the feeder topology and the selected combination satisfies the final feasible-set rule.

For M3, the selected CVI ranks are b28:r13, b22:r21, and b5:r22 in IEEE 33, and b47:r5, b71:r7, and b48:r8 in IEEE 123.

The selected M3 buses include global CVI ranks 13, 21, and 22 in IEEE 33 because the compact-feeder rule intentionally diversifies the candidate set by depth. In IEEE 123, the selected buses remain among the high global CVI ranks because pure CVI screening is retained for the larger feeder. Fig. 3 explains this difference. In IEEE 33, high-CVI candidates are concentrated toward deeper feeder locations, whereas the selected stratified candidates intentionally include near and middle-depth locations. In IEEE 123, the high-CVI candidates are already sufficiently useful under the large-feeder policy, so pure CVI screening is retained.

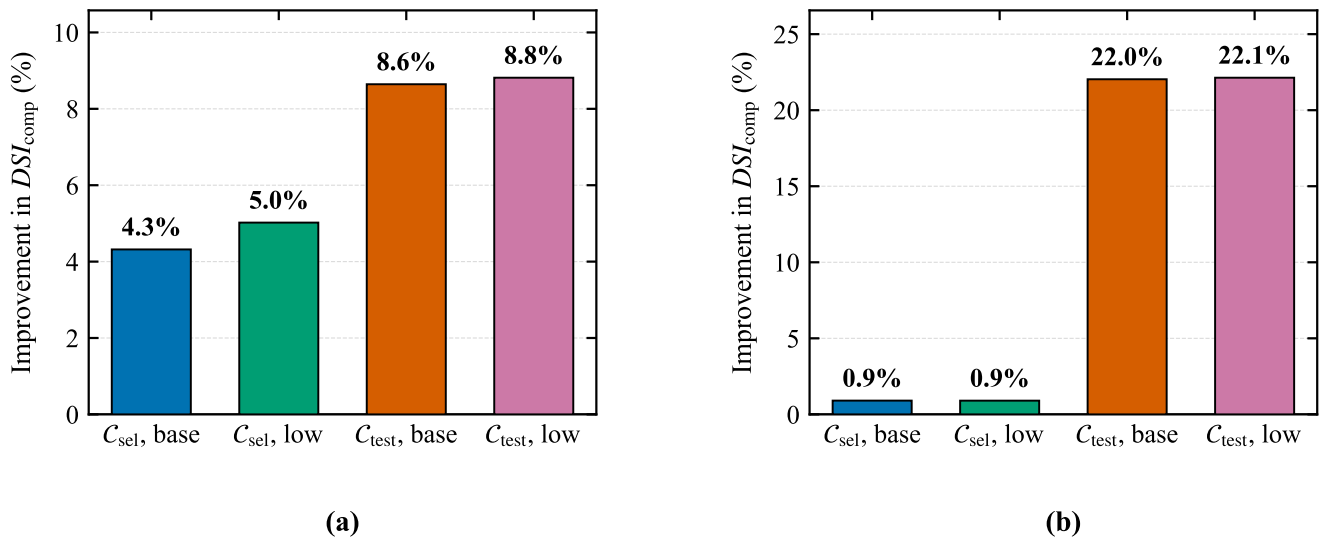
VII. RESULTS

A. MAIN RESPONSE-QUALITY RESULTS

Table 4 reports the main response-quality comparison. Lower values indicate better response quality. On IEEE 33, M3 improves DSI_{comp} over M2 by 4.3% and 5.0% on C_{sel} under base and low inertia. On C_{test} , the improvements increase to 8.6% and 8.8%. The topology-diversified policy improves the

TABLE 4. Main response-quality comparison and robust selection-objective values.

System	Method	DSI_{comp} ($C_{sel}, base$)	DSI_{comp} (C_{sel}, low)	DSI_{comp} ($C_{test}, base$)	DSI_{comp} (C_{test}, low)	DSI_{max} ($C_{sel}, base$)	DSI_{max} (C_{sel}, low)	Robust DSI_{max} sel. obj.
IEEE 33	M1: GFL-only	4.477	6.387	1.841	2.425	6.580	9.552	9.552
IEEE 33	M2: Topology-only distributed	1.505	1.653	0.671	0.658	2.168	2.443	2.443
IEEE 33	M3: Proposed	1.440	1.570	0.613	0.600	2.068	2.313	2.313
IEEE 33	CVI-top3 (unconstrained)	2.327	2.663	1.011	1.058	3.363	3.920	3.920
IEEE 123	M1: GFL-only	2.081	2.928	0.658	0.827	3.573	5.094	5.094
IEEE 123	M2: Topology-only distributed	0.994	1.108	0.413	0.384	1.685	1.906	1.906
IEEE 123	M3: Proposed	0.985	1.098	0.322	0.299	1.680	1.900	1.900
IEEE 123	CVI-top3 (unconstrained)	1.023	1.148	0.366	0.339	1.759	1.998	1.998

**FIGURE 4.** M3 improvement over M2 in DSI_{comp} across selection and hold-out sets for IEEE 33 and IEEE 123. Positive values indicate that M3 has lower DSI_{comp} than M2; (a) IEEE 33 and (b) IEEE 123.**TABLE 5.** Robust objective, all-feasible distribution, and current-limited power-flow feasibility summary.

System	$ \Pi $	Robust DSI_{max} objective	M3 DSI_{max} obj.	M3 rank	All-feasible mean $\pm$ std	Median	5–95 pct.	Max-current event	Bus	Max. $I_{pu,b}$
IEEE 33	68	Robust DSI_{max}	2.313	1/68	2.734 $\pm$ 0.215	2.753	2.378–3.163	L28-29	b5	0.9244
IEEE 123	84	Robust DSI_{max}	1.900	1/84	2.106 $\pm$ 0.123	2.104	1.946–2.341	LS+15@b96	b71	0.9172

IEEE 33 response relative to the topology-only distributed baseline, while the pure-CVI diagnostic remains inferior for this compact feeder.

For IEEE 123, M3 improves DSI_{comp} over M2 by 0.9% and 0.9% on the selection set and by 22.0% and 22.1% on the hold-out set. The in-sample gain is modest, so the claim should not be overstated as a large universal improvement. The stronger hold-out gain indicates that the high-CVI placement is especially helpful for the particular hold-out disturbances considered in this study. This is a useful planning result because the hold-out events are not used by the optimizer. Because the IEEE 123 hold-out result is computed

over five events, the 22.0%/22.1% improvement should be interpreted as evidence for the tested disjoint events rather than as a population-level estimate over all possible feeder contingencies. A larger Monte Carlo event set would be required to quantify statistical confidence intervals.

Fig. 4 summarizes these percentage improvements. The figure also shows why one should not claim that one policy dominates all feeders in the same way. The compact feeder obtains a moderate and consistent improvement after topology diversification. The large feeder obtains a small in-sample improvement and a larger improvement on the disjoint hold-out events considered in this study. The defensible

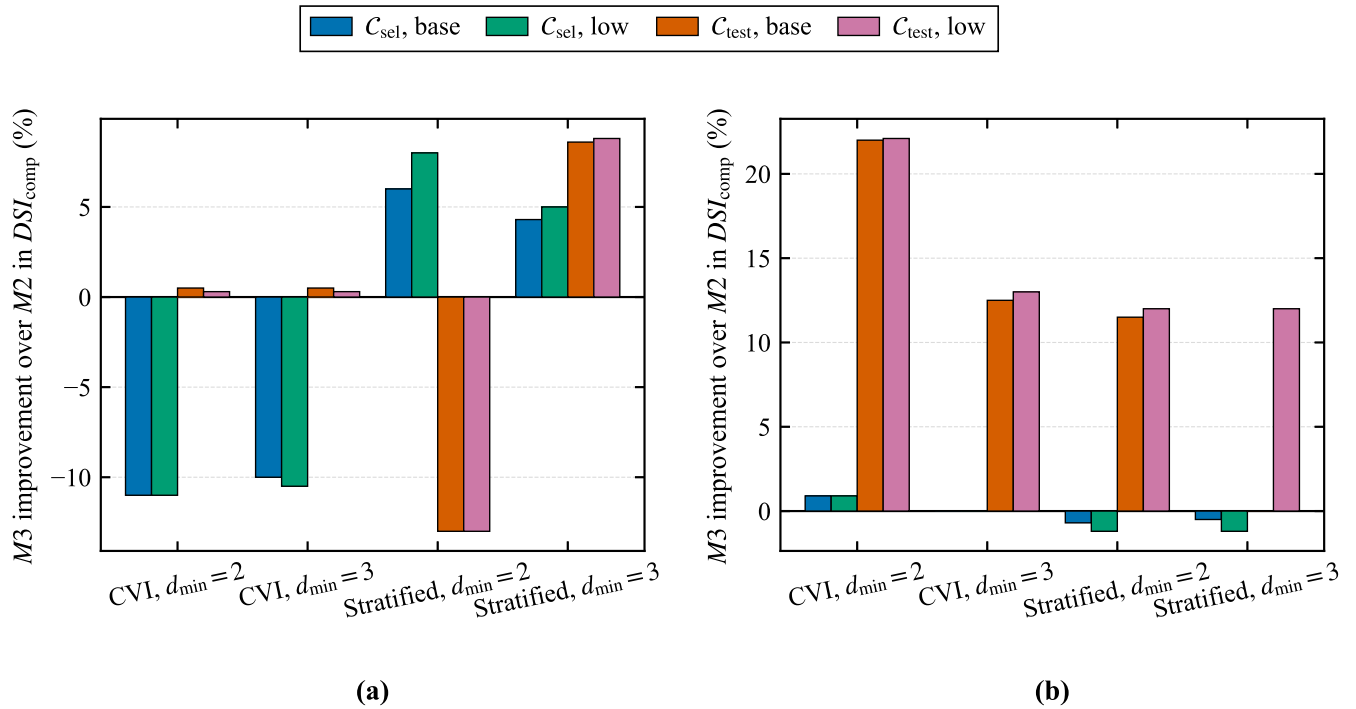


FIGURE 5. Screening-policy and minimum-distance ablation for IEEE 33 and IEEE 123. The ablation is reported for transparency; the final rule is pre-specified as a two-feeder case-study policy and is not selected after reading these bars. Panels (a) and (b) correspond to IEEE 33 and IEEE 123, respectively.

TABLE 6. Quantitative validation checks retained with the simulation records.

Validation item	IEEE 33	IEEE 123
Hold-out overlap, $ C_{\text{sel}} \cap C_{\text{test}} $	0	0
M3 minimum pairwise hop distance (required $d_{\min}$)	4 hops (3)	2 hops (2)
Maximum current proxy / limit	0.9244 / 1.2	0.9172 / 1.2
Current-limited PF feasibility record	infeasible snapshots = 0	infeasible snapshots = 0
Current-limited PF convergence	verified in retained records	verified in retained records
Active graph radiality	$ \mathcal{E} = 32 = 33 - 1$	$ \mathcal{E} = 124 = 125 - 1$
DSI_{comp} recomputation tolerance	max abs. error $< 10^{-6}$	max abs. error $< 10^{-6}$
Contingency-definition validity	invalid records = 0	invalid records = 0

claim is therefore not that M3 always produces large gains, but that the proposed workflow produces internally verified planning-stage improvements under the tested assumptions.

B. ABLATION OF SCREENING POLICY AND DISTANCE CONSTRAINT

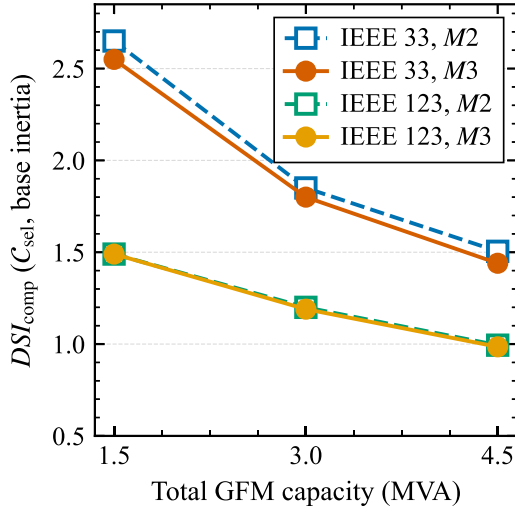
Fig. 5 reports the screening-policy ablation. For IEEE 33, pure CVI with $d_{\min} = 2$ or $d_{\min} = 3$ gives negative in-sample improvement relative to M2 under both inertia settings. Stratified screening with $d_{\min} = 2$ improves in-sample performance, but its hold-out performance is worse than M2. Stratified screening with $d_{\min} = 3$ is the only evaluated policy that improves both selection and hold-out sets. This pattern supports the compact-feeder policy: the issue is not only electrical weakness but also spatial coverage.

For IEEE 123, pure CVI with $d_{\min} = 2$ gives the strongest in-sample performance among the reported

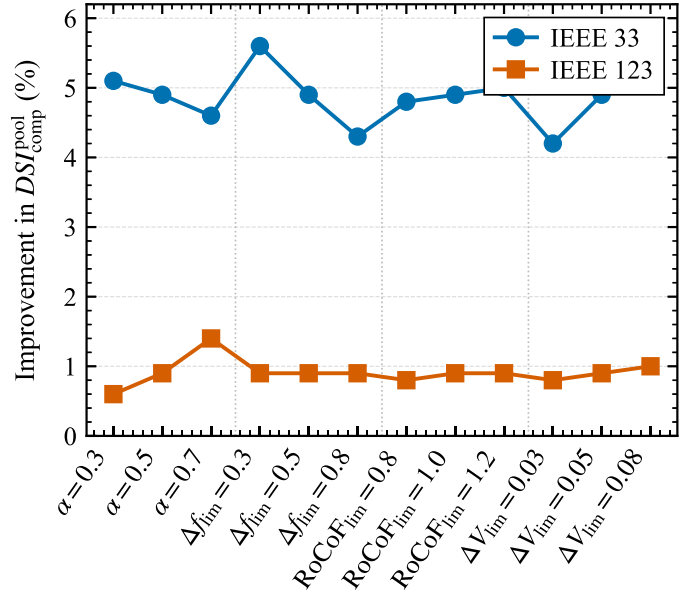
policies. Increasing the distance constraint to $d_{\min} = 3$ slightly weakens in-sample performance, and stratified screening is also weaker in-sample. This supports the large-feeder branch of the policy: forcing stratification is not automatically beneficial when the high-CVI buses already provide useful coverage.

C. VALIDATION, FEASIBLE-SET COMPARISONS, AND CURRENT CAPABILITY

Table 5 reports the robust selection objective, all-feasible objective distribution, voltage-adjusted current proxy, and distance-sensitivity result. The robust $DSI_{\text{max}}^{(\ell)}$ objective denotes $\max_{\ell \in \{\text{base}, \text{low}\}} DSI_{\text{max}}^{(\ell)}$ on C_{sel} . The all-feasible distribution is evaluated using the same robust base/low-inertia DSI_{max} objective, the same screened candidate set, and the same distance constraint as M3. Table 5 also reports the current-limited power-flow feasibility check. The selected



(a)



(b)

FIGURE 6. Capacity and DSI-normalization sensitivity. The capacity sweep evaluates the fixed M2 and M3 bus sets at different ratings without re-optimizing GFM penetration, and the DSI sweep evaluates whether M3 remains better than M2 under local changes in normalization constants and α ; (a) fixed-bus capacity diagnostic and (b) DSI-normalization sensitivity.

placements converge under the imposed 1.2 pu voltage-adjusted current-proxy constraint, and the largest recorded proxy values are 0.9244 pu and 0.9172 pu for IEEE 33 and IEEE 123, respectively. The reported maximum-current column gives the largest $I_{pu,b}$ over the selected GFM buses and evaluated post-event snapshots. This result should be interpreted as feasibility under the imposed planning constraint, not as proof of dynamic current-limiter stability. For the selected placements, the current proxy is not binding under the declared 1.2 pu limit and 70% active dispatch. Therefore, the current check is interpreted here as a feasibility check rather than as the primary discriminator among candidate placements. In the event labels, L28-29 denotes a line outage between buses 28 and 29, and LS+15@b96 denotes a +15% load-step event at bus b96.

The feasible-set comparison is deliberately strict. It enumerates every placement from the same screened candidate set used by M3 and enforces the same d_{min} . This means the comparison is not weakened by drawing from irrelevant buses, violating the topology constraint, or sampling only a small subset of feasible alternatives. The reported rank and distribution show where the selected placement sits within the entire screened feasible set. The result is not a proof of global optimality over every possible bus in the feeder, but it is a meaningful verification of the proposed optimizer inside the declared candidate envelope.

The distance-sensitivity sweep is also important. A common mistake is to compare different distance settings without re-optimizing after the feasible set changes. Here, each d_{min} value is re-optimized. The reported monotonicity check

confirms that tightening the distance constraint does not produce an impossible logical pattern in the feasible objective sequence. The sweep is retained as a supporting verification record rather than expanded into another main-text table, keeping the paper readable while preserving traceability within the retained records.

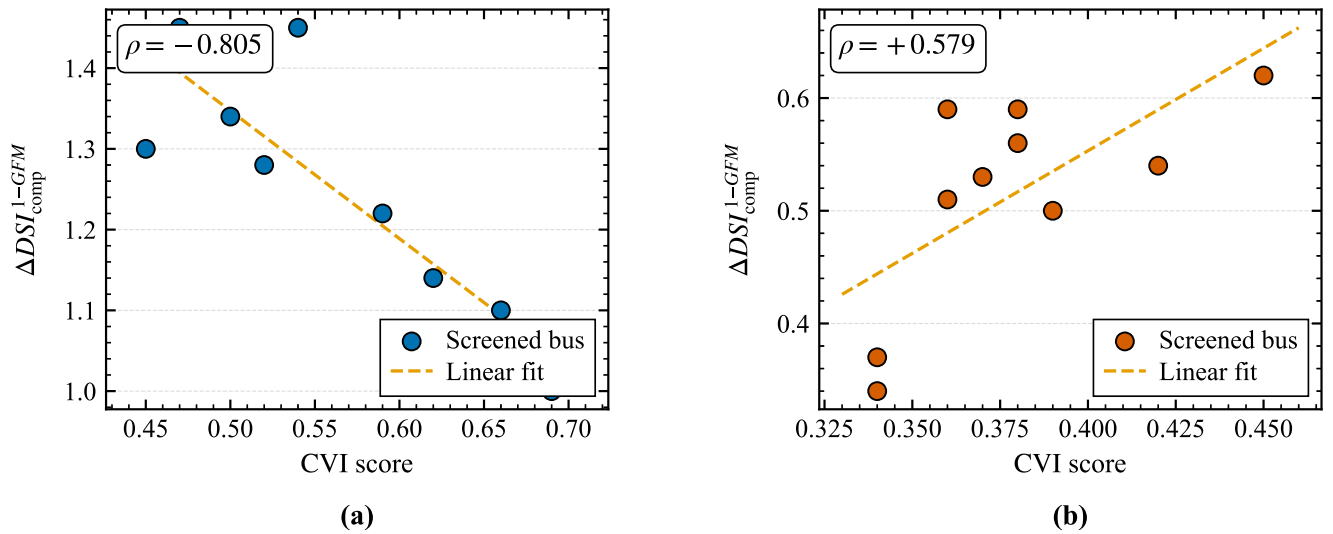
Table 6 reports the validation checks in quantitative form rather than hiding them behind a single pass/fail entry. These checks cover contingency-record validity, recomputation of DSI_{comp} , current-limited power-flow feasibility, hold-out separation, distance-constraint satisfaction, active-graph radially, and equation normalization. The values are taken from the retained validation records. Items without a complete scalar percentage, such as current-limited power-flow convergence over all internal solver calls, are reported as verified in retained records rather than assigned a fabricated percentage.

D. CVI-WEIGHT PERTURBATION DIAGNOSTIC

The CVI weights are fixed at (0.4, 0.3, 0.3) for SCR, voltage sensitivity, and local load concentration. A placement method should not depend on an extremely fragile weight choice, so the retained verification records include a one-at-a-time perturbation diagnostic. Each CVI component weight is perturbed upward and downward, and the overlap of the screened candidate list is recorded. In the current results, the top candidate set is unchanged for both feeders under the tested perturbations, producing a Jaccard overlap of 1.0 in the generated beta-perturbation records.

TABLE 7. DSI-normalization sensitivity values based on the pooled selection-event metric DSI_{comp}^{pool} .

Sensitivity case	IEEE 33 M2	IEEE 33 M3	Improvement	IEEE 123 M2	IEEE 123 M3	Improvement
$\alpha = 0.3$	1.966	1.865	5.1%	1.426	1.418	0.6%
$\alpha = 0.5$	1.648	1.566	4.9%	1.106	1.096	0.9%
$\alpha = 0.7$	1.330	1.268	4.6%	0.786	0.775	1.4%
$\Delta f_{lim} = 0.3$	2.269	2.141	5.6%	1.514	1.501	0.9%
$\Delta f_{lim} = 0.5$	1.648	1.566	4.9%	1.106	1.096	0.9%
$\Delta f_{lim} = 0.8$	1.298	1.243	4.3%	0.877	0.869	0.9%
$RoCoF_{lim} = 0.8$	1.761	1.676	4.8%	1.166	1.156	0.8%
$RoCoF_{lim} = 1$	1.648	1.566	4.9%	1.106	1.096	0.9%
$RoCoF_{lim} = 1.2$	1.572	1.493	5.0%	1.066	1.057	0.9%
$\Delta V_{lim} = 0.03$	1.687	1.617	4.2%	1.173	1.164	0.8%
$\Delta V_{lim} = 0.05$	1.648	1.566	4.9%	1.106	1.096	0.9%
$\Delta V_{lim} = 0.08$	1.626	1.538	5.4%	1.069	1.058	1.0%

**FIGURE 7.** CVI score versus single-GFM improvement in DSI_{comp} for IEEE 33 and IEEE 123. The IEEE 33 panel illustrates why CVI alone is insufficient for compact feeders; the final placement is selected by multi-GFM robust DSI_{max} optimization. Panels (a) and (b) correspond to IEEE 33 and IEEE 123, respectively.

This diagnostic should not be overstated as a full uncertainty analysis. It only checks local sensitivity around the selected CVI weights and does not test every possible weight vector. Its value is narrower but still useful: it shows that the reported placement is not created by a knife-edge CVI ranking in the present study. A broader future analysis could sample CVI weights, load levels, and line parameters jointly, then report confidence intervals for the selected buses and response metrics.

Fig. 6 adds two additional planning sensitivities. The capacity panel evaluates the fixed M2 and M3 bus sets at 0.5, 1.0, and 1.5 MVA per unit; it is not a capacity-optimization study, but it shows whether the selected M3 locations remain favorable relative to M2 as the fixed converter rating changes. The capacity sweep does not re-optimize the siting decision. The DSI-normalization panel perturbs α , Δf_{lim} , $RoCoF_{lim}$, and ΔV_{lim} . The M3 improvement remains positive across the tested perturbations, supporting the use of DSI as a relative planning metric for this case study. Fig. 6 details whether the selected M3 placement remains

favorable under fixed-capacity and DSI-normalization perturbations.

Table 7 reports the corresponding numerical values using the pooled selection-event sensitivity metric DSI_{comp}^{pool} defined in (15), not the base/low separated DSI_{comp} values reported in Table 4. Positive improvement means that M3 has lower DSI_{comp}^{pool} than M2. The exact numerical variations under different DSI-normalization weights in Table 7 support the visual sensitivity trends illustrated in Fig. 6. The selected M3 placement retains positive improvement over M2 under all tested perturbations.

E. CVI-RESPONSE DIAGNOSTICS AND EVENT-LEVEL DISTRIBUTIONS

Fig. 7 shows the relationship between CVI score and single-GFM response improvement. For the single-GFM diagnostic, the improvement at candidate bus b is defined as

$$\Delta DSI_{comp}^{1-GFM}(b; C) = DSI_{comp}(\emptyset; C) - DSI_{comp}(\{b\}; C), \quad (19)$$

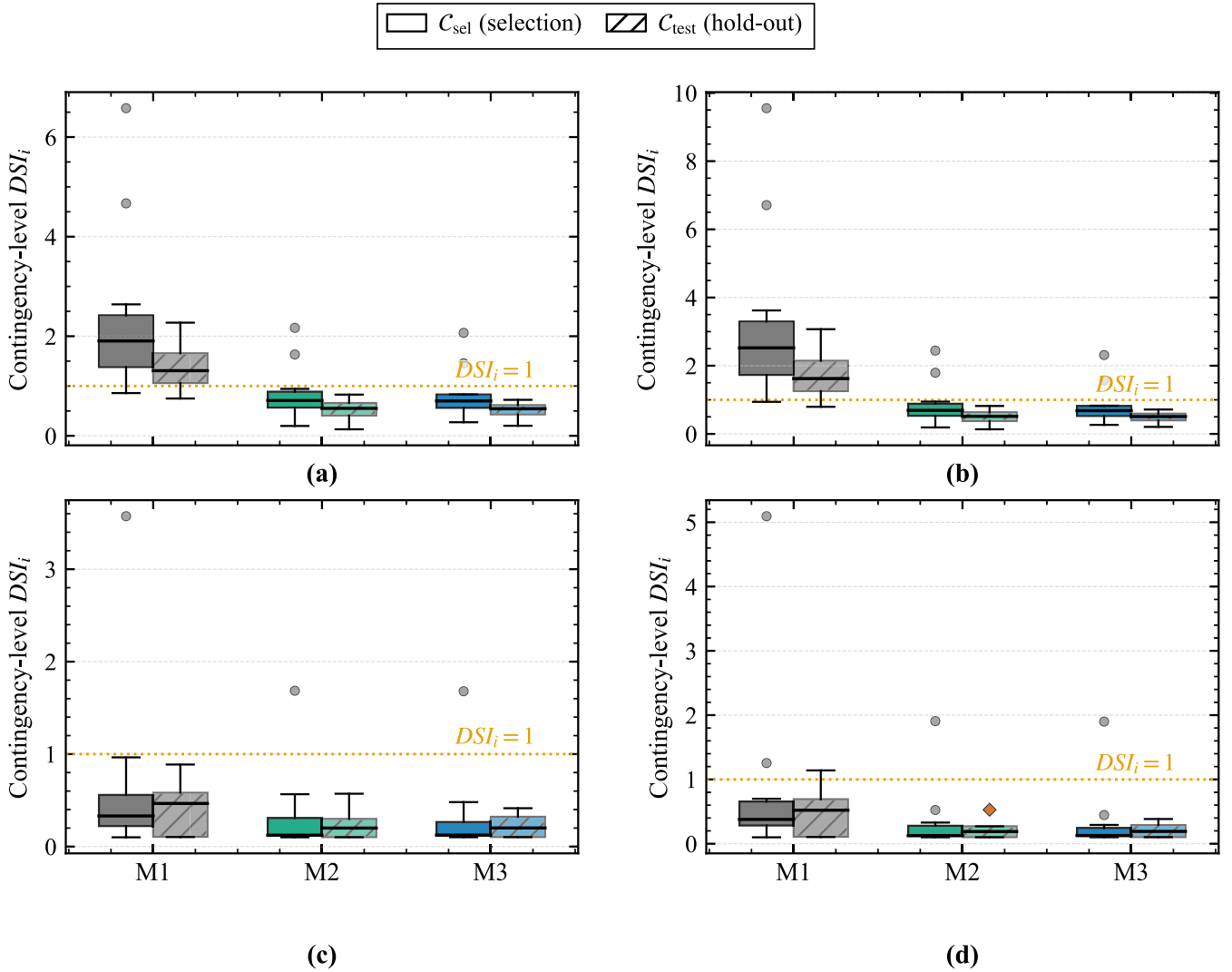


FIGURE 8. Contingency-level DSI distributions for selection and hold-out sets in IEEE 33 and IEEE 123, separated by feeder and inertia case for readability. The dotted line marks $DSI_i = 1$. These plots expose event-level behavior that can be hidden by a single average metric; (a) IEEE 33 base inertia, (b) IEEE 33 low inertia, (c) IEEE 123 base inertia, and (d) IEEE 123 low inertia.

where $\emptyset$ denotes the GFL-only case and $\{b\}$ denotes one GFM installed at bus b . Therefore, a positive value means that the single-GFM case lowers DSI_{comp} relative to the no-GFM case. The diagnostic in Fig. 7 is computed on the base-inertia selection-event set and is used only as a diagnostic, not as a primary evidence source for the final placement. The scatter should be interpreted carefully. CVI is a screening feature, not a complete surrogate for multi-GFM dynamic response. The negative single-GFM correlation in compact IEEE 33 indicates that local vulnerability and single-device improvement are not equivalent. In this feeder, high CVI values are concentrated in a downstream cluster, whereas robust three-GFM performance also requires spatial coverage. This is why CVI is used only to form a candidate envelope and is followed by distance-constrained multi-GFM enumeration. CVI is therefore not used to select the final placement by itself. In IEEE 33, CVI reduces the effective

candidate space from 32 non-substation buses to 10 screened buses, making exact multi-GFM enumeration tractable while the selected placement remains rank 1 within the screened feasible set. The single-GFM diagnostic highlights why the dynamic multi-GFM layer is necessary, especially in compact radial feeders where local vulnerability and spatial coverage can conflict. In IEEE 123, the high-CVI set remains useful under the large-feeder policy, but the final placement is still determined by multi-GFM robust DSI_{max} evaluation.

Fig. 8 reports event-level DSI distributions for the main methods under both base and low inertia. These distributions complement the composite table because they show whether the improvement is broad or driven by a few events. Fig. 8 unpacks the contingency-level DSI distributions so that the composite-index improvements are not interpreted as being driven only by a single averaged value. The M1 case is consistently worse because no GFM support is available. M2

TABLE 8. Correlation of the planning DSI with conventional frequency-stability indicators (pooled over all events, both placements and inertia cases), and broad-based improvement of the proposed M3 placement over the topology-only baseline M2 in each constituent indicator.

System	$R^2(\text{DSI}, \Delta f)$	$R^2(\text{DSI}, \text{RoCoF})$	$R^2(\text{DSI}, t_{\text{settle}})$
IEEE 33	0.98	0.92	0.50
IEEE 123	0.98	0.94	0.78
<i>M3 vs M2 improvement (%) per indicator</i>			
System	$ \Delta f $	RoCoF	t_{settle}
IEEE 33	+9.3	+4.1	+7.9
IEEE 123	+7.4	+2.9	+15.9

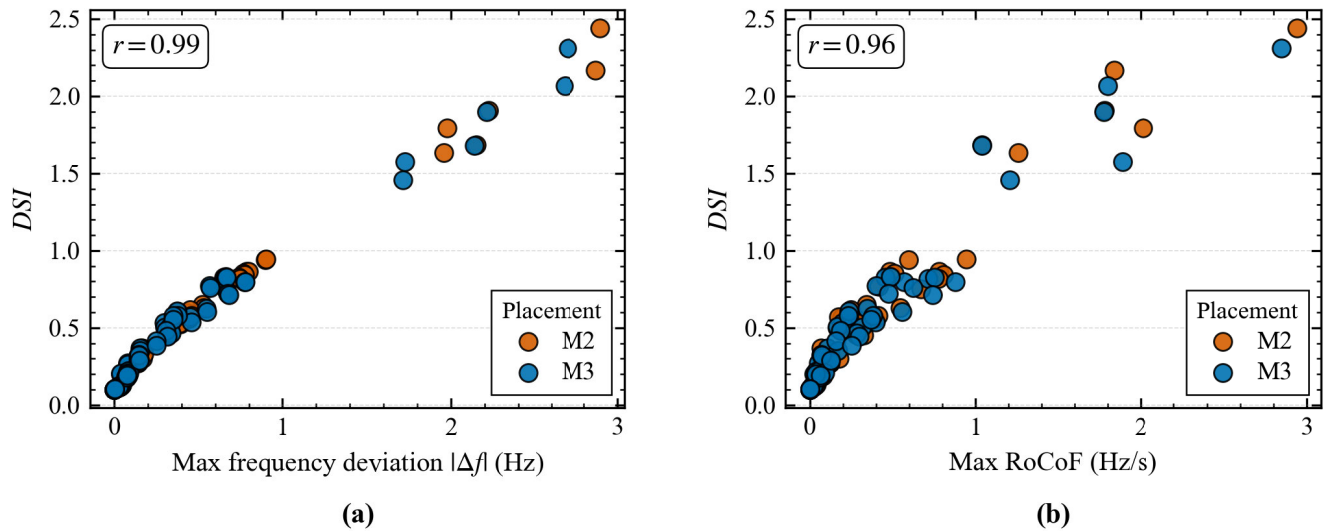


FIGURE 9. Planning DSI versus conventional frequency-stability components for every evaluated event in both feeders and both placements (M2 and M3). Panels (a) and (b) show the DSI versus the maximum frequency deviation $|\Delta f|$ and versus the maximum RoCoF, respectively. The association provides an internal-consistency check: changes in the composite DSI are accompanied by changes in its principal frequency-response components. It does not replace independent EMT-level validation.

provides a strong baseline. M3 then reduces the upper part of the event-level distribution under the tested conditions, especially in the hold-out events of IEEE 123. The dotted $\text{DSI}_i = 1$ reference line is useful for identifying events with normalized response severity above the nominal planning threshold.

F. VALIDATION OF THE SCREENING METRIC, METHOD COMPARISON, AND STATISTICAL ROBUSTNESS

This subsection provides additional verification of the screening metric, method comparison, and statistical robustness. First, to confirm that the reduced-order screening index reflects conventional stability behavior, the per-event DSI is decomposed against the maximum absolute frequency deviation, rate of change of frequency, and settling time. Table 8 shows that the DSI is strongly associated with the maximum absolute frequency deviation and rate of change of frequency, while its association with settling time is moderate in the IEEE 33 case ($R^2 = 0.50$) and stronger in the IEEE 123 case ($R^2 = 0.78$); the proposed placement nonetheless improves each of these constituent indicators relative to

the topology-only baseline rather than trading one against another. Fig. 9 visualizes this association across all evaluated events.

Second, to position the method against alternative selection rules, Table 9 compares the proposed placement with a grid-following case, the topology-only baseline, an unconstrained vulnerability-ranked selection, and a thirty-trial random baseline. The proposed placement attains the lowest composite index among all evaluated methods on the selection set and among the methods with reported hold-out results, and in the compact feeder the unconstrained vulnerability-only selection is in fact worse than the topology baseline, which supports the need for the topology-diversification step.

Third, Table 10 reports robustness of the selected placement to the influence-radius parameter and to the hop-weighting form. The selected buses are invariant across the tested radius range and identical under exponential and inverse-hop weighting in both feeders. On IEEE 123 the selection-set improvement is small and slightly negative at short radii, whereas the hold-out improvement remains large (15.9–22.5%, last column of Table 10), so the placement

TABLE 9. Method comparison of DSI_{comp} (lower is better, base inertia): GFL-only (M1), topology-only (M2), the proposed method (M3), an unconstrained CVI/sensitivity-based selection (CVI-top3), and a 30-trial random baseline. M3 yields the lowest DSI_{comp} among all evaluated methods, and the lowest among the methods for which hold-out results are reported.

System	Method	C_{sel}	C_{test}
IEEE 33	M1	4.477	1.841
IEEE 33	M2	1.505	0.671
IEEE 33	M3	1.440	0.613
IEEE 33	CVI-top3	2.327	1.011
IEEE 33	Random (best/30)	2.345	–
IEEE 123	M1	2.081	0.658
IEEE 123	M2	0.994	0.413
IEEE 123	M3	0.985	0.322
IEEE 123	CVI-top3	1.023	0.366
IEEE 123	Random (best/30)	1.946	–

TABLE 10. Robustness of the selected placement. Top: the selected M3 buses are invariant across the topological-influence radius $\tau \in \{8, 10, 12, 16, 20\}$; only the improvement magnitude varies, and for IEEE 123 both the selection-set and hold-out (h.o.) M3-over-M2 improvements (%) are reported. Bottom: the selected M3 buses are also invariant to the hop-weighting form (exponential vs. inverse-hop).

τ	IEEE 33 M3	IEEE 123 M3	IEEE 33 sel. imp. %	IEEE 123 sel. imp. %	IEEE 123 h.o. imp. %
8	(5, 22, 28)	(47, 48, 71)	+5.4	–2.1	+21.3
10	(5, 22, 28)	(47, 48, 71)	+4.9	–0.1	+22.5
12	(5, 22, 28)	(47, 48, 71)	+4.3	+1.0	+22.2
16	(5, 22, 28)	(47, 48, 71)	+3.2	+1.7	+18.7
20	(5, 22, 28)	(47, 48, 71)	+2.5	+1.9	+15.9

Hop-weighting-form ablation (selected M3 buses)		
Weighting	IEEE 33 M3	IEEE 123 M3
exponential	(5, 22, 28)	(47, 48, 71)
inverse-hop	(5, 22, 28)	(47, 48, 71)

is robust on the hold-out events even though the in-sample margin is thin; the exponential form is retained because it produces sharper locality, and an impedance-based weighting is not used because the reconstructed feeder employs grouped representative line impedances whose approximation error would propagate directly into such a weighting.

Finally, Table 11 quantifies statistical robustness through a nonparametric bootstrap confidence interval and an event leave-one-out range. The bootstrap interval is strictly positive only for the IEEE 33 selection set; the IEEE 123 selection-set interval and both hold-out intervals include zero, so those improvements are reported as descriptive case-study results rather than as population-level evidence of statistical significance.

VIII. DISCUSSION

A. ENGINEERING INTERPRETATION

The results support a planning-stage claim with bounded scope. First, IEEE 33 shows moderate but consistent improvement relative to the topology-only distributed baseline across base/low inertia and selection/hold-out sets. Second, both evaluated cases satisfy the current-limited power-flow feasibility check under the declared 1.2 pu voltage-adjusted current-proxy constraint. This should be read as feasibility under an imposed planning constraint, while dynamic

current-limiter stability remains outside the reduced-order model.

The IEEE 123 result should also be interpreted in light of its high converter-to-load ratio. Because the reduced positive-sequence load is 1.745 MW and the installed GFM rating is 4.5 MVA, the case tests the siting workflow under a high-GFM-penetration reduced planning condition. It should not be generalized as a typical penetration level for the original phase-domain IEEE 123 benchmark.

The physical interpretation is that CVI identifies buses where local support may be valuable, while the topology rule controls whether that support is concentrated or distributed. In the compact feeder, too much concentration weakens performance on the disjoint hold-out events considered here. In the larger feeder, the high-CVI set itself is sufficient, and additional stratification can discard useful buses. This is a more nuanced and defensible conclusion than claiming that one ranking rule universally solves the placement problem.

The role of DSI_{max} also deserves emphasis. The selected placement minimizes the worst event-level response on C_{sel} , not merely the average response. This is important for distribution planning because a placement that improves most events while leaving one severe disturbance unresolved may not be acceptable. Reporting DSI_{comp} alongside DSI_{max} provides both mean-worst and worst-case views.

TABLE 11. Statistical robustness of the M3-over-M2 DSI_{comp} improvement (base inertia): nonparametric bootstrap 95% confidence interval (2000 resamples over events) and event leave-one-out (LOO) range. The bootstrap interval is strictly positive only for the IEEE 33 selection set; the IEEE 123 selection-set interval and both hold-out intervals include zero and are reported as descriptive results.

System	Set	Imp. %	95% CI	LOO range %
IEEE 33	Csel	4.3	[2.4, 9.2]	[4.0, 8.4]
IEEE 33	Ctest	8.7	[-5.5, 11.5]	[0.5, 9.9]
IEEE 123	Csel	1.0	[-1.9, 15.0]	[0.5, 12.9]
IEEE 123	Ctest	22.2	[-6.9, 25.7]	[-6.2, 24.0]

B. REPRODUCIBILITY AND VERIFICATION RECORDS

The study avoids several practices that would weaken reproducibility and protocol consistency. The final policy is not named after the test feeders and does not use the hold-out events for placement selection. The candidate list reports both global CVI rank and screening rank, which prevents the reader from mistaking the final locations for a simple top-CVI result. The feasible-set comparison uses the same candidate set and distance rule as M3, so the comparison is not artificially easy. Negative or mixed ablation results are included in the paper rather than omitted.

Verification-record retention is treated as part of the study protocol. The retained verification records include selected candidates, main performance, current capability, validation gates, feasible-set objectives, distance sweeps, screening-policy ablation, contingency definitions, and equation consistency checks. The manuscript uses only a compact subset of those records in the main text; these records are available from the corresponding author upon reasonable request.

To further address robustness to the choice of selection events and to operating-condition uncertainty, two additional analyses are reported. First, a leave-one-out re-optimization removes each selection event in turn and re-selects the placement; the selected buses are invariant in every run (Table 12), so the placement does not depend on any single selection event. Second, a Monte Carlo study perturbs the feeder load level over sixty trials per feeder and re-selects the placement (Fig. 10, Table 13). For the IEEE 33 feeder the nominal placement is re-selected in 96.7% of trials with a median improvement of 4.28% (95% percentile interval [2.79, 4.37]). For the IEEE 123 reconstruction the median improvement is 1.35% (interval [1.11, 3.27]); the selection remains within an adjacent mid-feeder zone (around buses 46–48, 63, and 71) while the exact bus indices are load-sensitive, and one of the sixty trials produced an anomalous large-negative value that is excluded by the percentile interval. This load-perturbation sensitivity analysis indicates that the placement is stable on the compact IEEE 33 feeder (re-selected in 96.7% of trials) but marginal on the larger IEEE 123 reconstruction (re-selected in only 1.7% of trials, with one large-negative outlier), consistent with the planning-stage scope of the method.

C. LIMITATIONS AND DEPLOYMENT PATH

The framework should be used as a screening tool. Before field deployment, several analyses are still required. The

simplified IEEE 123-node-derived reconstruction should be replaced by a phase-domain model with exact line codes, regulators, capacitors, transformers, and load models. The mixed DSI construction should also be replaced or supplemented by simulations that explicitly capture voltage transients rather than only post-event power-flow snapshots. Detailed converter controllers and current limiters should be simulated, including saturation, recovery, and protection interactions. Protection coordination, islanding logic, communication assumptions, and cyber-physical failure modes should be evaluated. Load and renewable uncertainty should be expanded beyond the finite event sets used here. The present reduced-order, positive-sequence planning screen does not by itself establish detailed converter, current-limiter, or unbalanced-feeder behavior. As the immediate next step, an operational-stage study is planned that validates a representative subset of placements with electromagnetic-transient and controller current-limiter models, extends the analysis to a three-phase unbalanced feeder representation, evaluates additional feeders with varying resistance-to-reactance ratios, meshed sections, and distributed-energy-resource penetration levels, and replaces the finite event sets with a stochastic scenario study reporting confidence intervals.

These limitations do not invalidate the present results; they define their scope. A planning-stage paper can make a valuable contribution if it is clear about what has been checked and what remains outside the model. The appropriate claim is that topology-diversified CVI screening with a separate current-proxy feasibility check can improve the tested response-quality metrics under the reduced-order assumptions used here.

D. SCOPE OF THE CLAIMS

The strongest defensible claim is comparative and scoped: under the stated planning model, the proposed M3 workflow improves the tested response-quality metrics relative to the topology-only distributed M2 baseline while satisfying the current-capability screen. It is also defensible to claim that compact feeders may require topology diversification even when CVI is used, because the IEEE 33 ablation shows that pure CVI is not sufficient in that case. For high-GFM-penetration IEEE 123, it is defensible to claim that pure CVI can remain effective when the high-CVI set already provides useful spatial coverage.

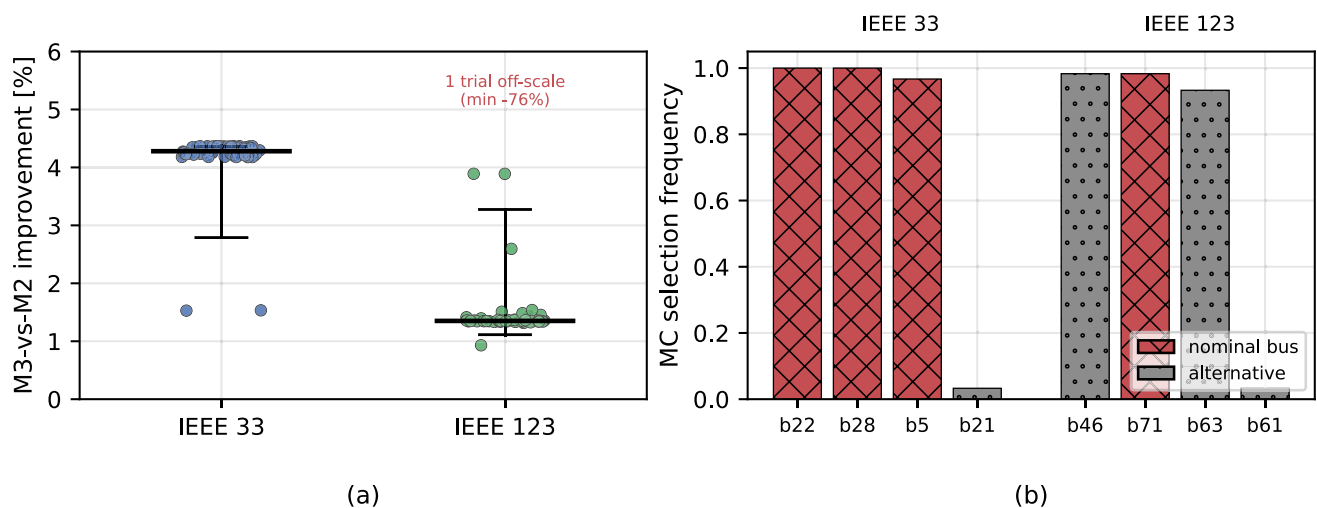
The paper should avoid stronger claims that are not supported by the present model. It should not state that the

TABLE 12. Leave-one-out (LOO) re-optimization over the selection-event set: the placement is re-selected with each event removed. The selected buses are invariant in every LOO run (Jaccard = 1.0), confirming that the placement does not depend on any single selection event.

System	Selected M3	LOO runs	Identical	Mean Jaccard
IEEE 33	(5, 22, 28)	12	12/12	1.00
IEEE 123	(47, 48, 71)	15	15/15	1.00

TABLE 13. Monte Carlo load-perturbation sensitivity of the M3-over-M2 DSI_{comp} improvement under random load-level perturbation (60 trials per feeder, selection set, base inertia). The median and a nonparametric 95% percentile interval are reported (robust to outliers); the minimum improvement and the number of negative trials are also listed to surface unfavorable outcomes. For IEEE 33 the nominal placement is re-selected in nearly all trials; for IEEE 123 the selection remains within an adjacent mid-feeder zone while the exact bus indices are load-sensitive.

System	Trials	Median imp. [%]	95% PI [%]	Nominal re-sel. [%]	Min imp. [%]	Neg. trials
IEEE 33	60	4.28	[2.79, 4.37]	96.7	+1.53	0
IEEE 123	60	1.35	[1.11, 3.27]	1.7	-76.39	1

**FIGURE 10.** Monte Carlo load-perturbation sensitivity under random load-level perturbation (60 trials per feeder). (a) Per-trial M3-over-M2 DSI_{comp} improvement (jittered points) with the median (thick bar) and 95% percentile interval (whisker); the vertical axis is clipped, so one IEEE 123 trial near -76% lies outside the view and is excluded by the percentile interval. (b) Bus selection frequency of the most-selected candidates; nominal buses are highlighted.

selected buses are globally optimal over all possible GFM sizes, controller parameters, operating points, or unbalanced conditions. It should not state that the IEEE 123 result is an exact benchmark result. It should not state that current-limiter stability has been proven. It should not generalize the feeder-size threshold as a universal standard. These boundaries make the paper more credible, not weaker, because they show that the simulation evidence and the written claims are aligned.

The resulting framing is a deterministic planning methodology with retained case-study verification records. The novelty is not only the numerical improvement; it is the internally verified structure around the placement decision.

IX. CONCLUSION

This paper presented a planning-stage framework for GFM inverter siting in radial distribution feeders. The method combines CVI screening, a pre-specified two-feeder

case-study policy, exact enumeration of feasible placements over the screened set, robust base/low-inertia DSI_{max} -based selection, disjoint hold-out checks, feasible-set comparisons, distance-sensitivity checks, screening-policy ablation, DSI and capacity sensitivities, and voltage-adjusted current-proxy feasibility checks. On the IEEE 33-bus feeder, the proposed placement improves DSI_{comp} over the topology-only distributed baseline by 4.3%/5.0% on the selection set and 8.6%/8.8% on the hold-out set under base/low-inertia cases. On the high-GFM-penetration simplified IEEE 123-node-derived reconstruction, the corresponding improvements are 0.9%/0.9% and 22.0%/22.1%. For the post-event power-flow snapshots evaluated in this study, the selected placements yield feasible current-limited power-flow solutions under the declared 1.2 pu voltage-adjusted current-proxy constraint. For the placements and dispatch evaluated here, the current proxy is non-binding, with maximum recorded proxy values of 0.9244 pu and 0.9172 pu against the 1.2 pu limit; the

current-limited power-flow check therefore serves as a feasibility check rather than as an active selection constraint in the present cases. The results support the proposed workflow as a deterministic planning-stage screening method under the stated reduced-order and positive-sequence assumptions. Detailed unbalanced feeder modeling, manufacturer-specific GFM controls, protection coordination, and dynamic current-limiter simulations remain necessary before deployment.

DATA AVAILABILITY STATEMENT

The event definitions, screened candidate lists, feasible-placement objective tables, validation-gate records, post-event current-proxy summaries, and additional simulation and sensitivity-analysis data records supporting the findings of this study are available from the corresponding author upon reasonable request.

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