

## Reliability-Based Topology Optimization of Frame Structures for Multiple Criteria Using SLSV Method\*

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### Abstract

The SLSV (single-loop-single-vector) method is modified for the reliability-based optimization problem with multiple reliability constraints. The design problem is formulated to minimize the structural volume of frame structure in terms of cross-sectional area of each frame element subjected to the two mode reliability constraints. The two mode reliability criteria consist of the mean compliance and mean eigenfrequency. The limit state functions are formulated as normalized form to achieve numerical stability of the SLSV method, because the functions are directly adopted as constraint conditions. That is a large difference from the conventional double-loop method, where the limit state functions do not appear in the optimization loop. Through numerical examples of 2-D and 3-D frame design problems, higher computational efficiency and sufficient reliability approximation accuracy by the SLSV method are demonstrated in comparison with the conventional double loop method that the mode reliabilities are evaluated by the first order reliability method (FORM) in each optimization step. Additionally, the importance of normalization of the limit state functions in the SLSV method is also demonstrated.

**Key words :** Reliability-Based Topology Optimization, SLSV Method, Ground Structure Approach, First Order Reliability Method, Frame Structures

### 1. Introduction

Reliability-based design optimization (RBDO) has been formulated as a double loop approach that the reliability evaluation loop by such as the first order reliability method (FORM) is nested inside of the optimization loop<sup>(1)</sup>. Such a double-loop algorithm requires huge computational costs, and hence a practical application of RBDO is almost prohibited.

Recently, several computational efficient algorithms were proposed to reduce computational cost of the RBDO problem. Chen *et al.*<sup>(2)</sup> proposed the single loop single vector (SLSV) method that enables single loop design optimization by replacing mode reliability constraints to equivalent deterministic constraints by using an approximate design point information. On the other hand, Wu and Wang<sup>(3)</sup> proposed serial single loop methods called SFA (Safety-Factor Approach). The SFA introduced the concept of applying a safety factor in reliability design problems to optimization methods, and used approximately equivalent deterministic constraints. Du and Chen<sup>(4)</sup> proposed the sequential optimization and reliability

\*Received 4 June, 2010 (No. T2-08-0692)  
Japanese Original: Trans. Jpn. Soc. Mech. Eng., Vol.75, No.751, C (2009), pp.719–726 (Received 8 Aug., 2008)  
[DOI: 10.1299/jcst.4.172]

assessment (SORA) method. The SORA method transforms a probabilistic design problem to an equivalent deterministic optimization problem which solves deterministic optimization after alternating the value of constraints of constraints shift by using information obtained from the reliability assessment. Yang and Gu<sup>(5)</sup> indicated that the SFA and SORA are conceptually identical, although they are presented independently. Also, Yang and Gu<sup>(5),(6)</sup> compared the computational efficiency and the reliability approximation accuracy of the SLSV, SORA and the other methods for several numerical examples.

Concerning the topology optimization problem with uncertainties, the reliability-based topology optimization (RBTO) has been presented in recent papers<sup>(7),(8)</sup>, and these papers show that the reliability aspect can be successfully applied to topology optimization using continuum mechanics. Allen *et al.*<sup>(9)</sup> also proposed RBTO to develop a compliant micro-electromechanical system mechanism. However, as far as authors' knowledge, the recent efficient RBDO method is not sufficiently investigated for the RBTO problems yet, except for Min and Bang<sup>(10)</sup> that the SLSV method is applied to the RBTO problem using continuum mechanics and demonstrated the computational efficiency.

For the topology optimization using discrete elements, part of the authors have proposed the RBTO for frame structure design in the conceptual design phase<sup>(11)</sup>. For the frame structure with a nonstructural mass, the system reliability under stiffness and eigenfrequency criteria is evaluated, where the applied load and nonstructural mass are treated as random variables. Then, the method is expanded to the problem with uncertainty of cross-sectional property<sup>(12)</sup>. The research shows that the reasonable reliability-based configurations are obtained for several numerical examples. However, it requires much computational time, because it adopts the conventional double-loop method. Particularly, it requires large number of the limit state function calls in the reliability analysis, because a cross-sectional property of each frame element is modeled as random variable.

In this study, the SLSV method is applied to the reliability-based topology optimization of a frame structure for multiple criteria to investigate the computational efficiency and the reliability approximation accuracy. Note that the SLSV method is mode reliability-based design approach, not the system reliability-based approach, that is different from the authors' previous studies<sup>(11),(12)</sup>.

On applying the SLSV, the limit state functions are normalized to achieve numerical stability for optimization. A limit state function is defined to distinguish the safety and failure regions by sign and the value of the function itself has no meaning. Additionally, the conventional double-loop method does not directly have limit state functions as constraint condition or objective functions, that appears only in the reliability analysis loop. Therefore, it is not required to consider the magnitude of the limit state function itself. On the other hand, the SLSV method uses the limit state functions as constraint conditions directly. It indicated that numerical condition of the limit state functions affects the convergence performance of RBTO.

## 2. Reliability-based topology optimization of frame structure

Consider that an elastic mechanical structure consisting of  $I$  frame elements and non-structural mass with uncertainty at a node is subjected to a uncertain static load  $f$  at point  $P_k$  and fixed at boundary  $\Gamma_d$ , as shown in Fig. 1. The design problem can be formulated to minimize the structural volume subject to the stiffness and vibration constraints in terms of each element cross-sectional area. When the applied load and structural mass with variations are modeled as random variables, the stiffness and eigenfrequency also have variations. Therefore, these constraints are modeled as reliability constraints such that the probability exceeding a specified criterion value is constrained.

In this study, the mode reliability-based design problem is formulated to minimize the structural volume subjected to the two mode reliability constraints as follows.

$$\text{Minimize: Volume}(\mathbf{d}) \quad (1)$$

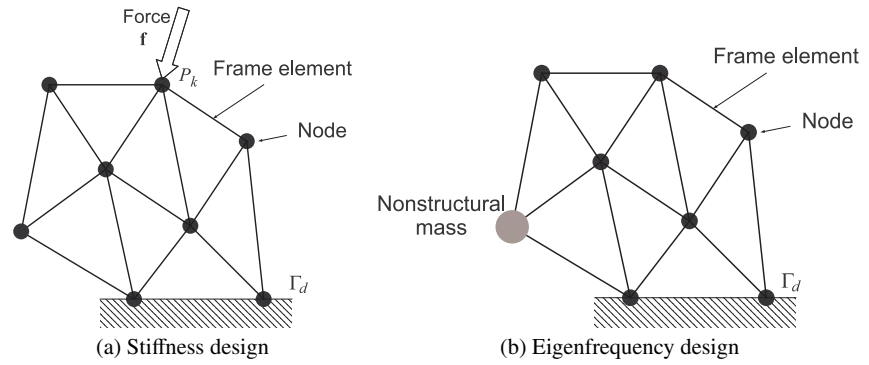


Fig. 1 Configuration of design domain.

$$\text{subject to: } P[g_j(\mathbf{d}, \mathbf{x}) \leq 0] \leq \Phi(-\beta_{T_j}), \quad (j = 1, 2)$$

$$\mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U$$

where  $\mathbf{d}$  is a design vector consisting of cross-sectional area of frame elements,  $\mathbf{d}^L$  and  $\mathbf{d}^U$  are lower and upper limits, respectively,  $\mathbf{X}$  indicates random variable consisting of applied force and nonstructural mass,  $\beta_{T_j}$  indicates the lower limit of the  $j$ -th mode reliability index, and  $\Phi(\cdot)$  is a standardized normal distribution function.

For the two reliability modes, one is the stiffness criterion, a mean compliance  $l$  and the other is the vibration criterion, a mean eigenfrequency  $\Lambda^{(13)}$ . The limit state functions are formulated as follows.

$$g_1(\mathbf{d}, \mathbf{x}) = \frac{l^U - l(\mathbf{d}, \mathbf{x})}{l^U} \quad (2)$$

$$g_2(\mathbf{d}, \mathbf{x}) = \frac{\Lambda(\mathbf{d}, \mathbf{x}) - \Lambda^L}{\Lambda^L} \quad (3)$$

where  $l^U$  indicates the upper limit of the mean compliance and  $\Lambda^L$  is the lower bound of the mean eigenfrequency.

The limit state function is generally defined to distinguish between the safety region ( $g > 0$ ) and the failure region ( $g < 0$ ) and the magnitude itself has no meaning. In the conventional double-loop method, the limit state functions do not appear in the optimization loop explicitly. That is, this kind of normalization has no effect on computation. On the other hand, in the SLSV method, the limit state functions explicitly appear in the optimization problem. Therefore, the limit state functions should be normalized in Eqs. (2) and (3) for numerical stability.

Note that the formulation in Eq. (1) is based on the mode reliability, not the system reliability<sup>(1)</sup>.

### 3. Formulation of topology optimization and limit state function

The optimization presented here is based on topology optimization techniques. The key ideas are the introduction of a fixed and extended design domain  $D$  that includes the original design domain  $\Omega_d$ , and the utilization of characteristic function below that indicates the existence of a frame element. Suppose that a fixed and extended design domain is composed of  $n$  frame elements. The existence of the  $i$ -th frame element is expressed by the following characteristic function:

$$\chi_i = \begin{cases} 1 & \text{if exists in } \Omega_d \\ 0 & \text{if does not exist in } \Omega_d \end{cases} \quad \text{for } i = 1, \dots, n \quad (4)$$

An optimal structure is obtained via this characteristic function when only necessary elements exist in the design domain  $\Omega_d$ . This approach is also termed the ground structure approach, where a set of fixed nodal points and some possible connections using discrete structural elements are first constructed, and an optimal configuration is then obtained by eliminating

unnecessary elements in the design domain using an optimization scheme. The optimization problem can thus be interpreted as a combinatorial problem that includes finite discontinuities that express the existence or non-existence of elements, and it is therefore not easily treated numerically. To overcome this difficulty, Equation (4) is approximated using the following equations, whose formulation is based on the concept of the density approach<sup>(14)</sup>.

$$\chi_i \approx \rho_{A_i}^p \quad (5)$$

$$0 \leq \rho_{A_i} \leq 1 \quad (6)$$

where  $\rho_{A_i}$  is a normalized design variable for the  $i$ -th element, and  $p$  is a penalization parameter emphasizing the influence of design variable. In order to achieve consistency between the normalized design variable and the actual design variable,  $p$  is set to 1. In cases where “exaggerated” optimal configurations are desired, which are composed of elements having higher and lower (but not intermediate) design variables, the penalization parameter can be set to 2 or 3. Using Eq. (5), the original combinatorial problem is replaced by an element distribution problem, and an optimal configuration can be obtained by eliminating unnecessary frame elements in the design domain using the proposed optimization scheme.

In this study, frame structures are considered where the frame elements support an axial load as well as bending and twisting moments. The cross-sectional area of the  $i$ -th element,  $A_i$ , is defined as follows.

$$A_i = \rho_{A_i}^p A_{\max} \quad (i = 1, \dots, I) \quad (7)$$

where  $A_{\max}$  is the maximum cross-sectional area and  $\rho_{A_i}$  is adopted as a normalized design variable. The other cross-sectional properties are easily obtained, assuming that the cross-sectional shape is a solid circle. The moment of inertia and polar moment of inertia are described in terms of the cross-sectional area as follows.

$$I_{y_i} = I_{z_i} = \frac{\pi d_i^4}{64} = \frac{A_i^2}{4\pi} \quad (8)$$

$$J_{x_i} = \frac{\pi d_i^4}{32} = \frac{A_i^2}{2\pi} \quad (9)$$

Using these parameters, we can construct a stiffness matrix  $\mathbf{K}_e$  for each element.

For the topology optimization, two criteria are introduced. One is the mean compliance, used for static criterion, the other is the mean eigenfrequency, used for dynamic criterion.

### 3.1. Mean compliance

Consider that an elastic mechanical structure consisting of  $I$  elastic frame elements is subjected to a static load  $\mathbf{f}$  at point  $P_k$  and fixed at boundary  $\Gamma_d$ , as shown in Fig. 1 (a). Body forces are assumed to be ignored for simplicity in the formulation. Let  $\mathbf{f}$  and  $\mathbf{v}$  be the load and displacement vectors, respectively. The mean compliance  $l$  is introduced as a measure of the stiffness at point  $P_k$ , expressed as follows<sup>(15),(16)</sup>.

$$l = \mathbf{f}^T \mathbf{v} = \mathbf{v}^T \mathbf{K} \mathbf{v} \quad (10)$$

where  $\mathbf{K}$  is the global stiffness matrix of the entire structure.

### 3.2. Mean eigenfrequency

Eigenfrequency is adopted as a dynamic criterion. Consider a frame structure consisting of  $I$  frame elements and a nonstructural mass, as shown in Fig. 1 (b). A vibration equation can be formulated as follows.

$$(\mathbf{K} - \lambda_m \mathbf{M}) \boldsymbol{\psi}_m = \mathbf{0}, \quad (m = 1, \dots, M) \quad (11)$$

where  $\mathbf{M}$  is the global mass matrix of the entire mechanical structure,  $\lambda_m$  ( $m = 1, 2, \dots, M$ ) is the  $m$ -th eigenvalue, and  $\boldsymbol{\psi}_m$  ( $m = 1, 2, \dots, M$ ) is the  $m$ -th eigenvector. The element mass matrix  $\mathbf{M}_e$  is obtained as following consistent mass matrix.

$$\mathbf{M}_e = \int_{V_e} \rho_e \mathbf{N}^T \mathbf{N} dV \quad (12)$$

where  $\rho_e$ ,  $N$  and  $V_e$  are element density, the shape function, and the element volume, respectively.

In usual mechanical design situations, the first eigenfrequency is maximized to avoid resonance phenomenon but optimum designs that focus on an eigenfrequency often encounter significant problems with mode shifting. To avoid the mode shifting problem, the following mean eigenvalue<sup>(13)</sup> is adopted.

$$\Lambda = \lambda_0 + \left( \sum_{m=1}^M \frac{w_m}{\lambda_m - \lambda_{0m}} \right)^{-1} \quad (13)$$

where  $w_m$ ,  $\lambda_0$  and  $\lambda_{0m}$  are the weighting factor, and design parameters, respectively.

During topology optimization under vibration criteria, local vibration modes that have no physical meaning will appear for excessively thin frame elements during the optimization search. In this study, Tcherniak's proposed method<sup>(17)</sup> is adopted to avoid such a phenomenon. When the design variable  $\rho_{A_i}$  is lower than the threshold value  $\rho_{thr}$ , the corresponding element mass matrix is regarded to zero, as follows.

$$M_e = \begin{cases} M_e & \text{if } \rho_{A_i} > \rho_{thr} \\ \mathbf{0} & \text{if } \rho_{A_i} \leq \rho_{thr} \end{cases} \quad (14)$$

where,  $\rho_{thr}$  is set to be 0.01 in this study. That is, the element mass matrix is set to zero, if the cross-sectional area is less than the threshold value,  $\rho_{thr}$ .

#### 4. SLSV method

For conventional reliability-based design optimization (RBDO) problem, the first-order reliability method (FORM) has been widely used for evaluating the mode reliability<sup>(1)</sup>. Since the FORM requires iterative calculation such as Rackwitz-Fiessler method<sup>(18)</sup> or nonlinear programming method, the RBDO is formulated as a nested loop calculation problem. Therefore, it is almost prohibitive to apply for a large-scale RBDO problem.

The single loop single vector (SLSV) method was developed to reduce the excessive computational cost of the double-loop problem<sup>(2)</sup>. The key idea is that the reliability constraint is converted into the deterministic constraint by using an approximated design point information.

In case that design vector  $\mathbf{d}$  is the mean value of random vector  $\mathbf{x}$  with normal distribution  $N(\mathbf{d}, \boldsymbol{\sigma})$ , the design point vector of the  $j$ -th mode  $\mathbf{x}_{(j)}^*$  is described in  $\mathbf{X}$ -space as shown in Fig. 2<sup>(2)</sup>.

$$\mathbf{x}_{(j)}^* = \mathbf{d} - \beta_j \boldsymbol{\sigma}^T \boldsymbol{\alpha}_j \quad (15)$$

$$\boldsymbol{\alpha}_j = \frac{\nabla g_j(\mathbf{d}, \mathbf{x}_{(j)}^*)}{|\nabla g_j(\mathbf{d}, \mathbf{x}_{(j)}^*)|} \quad (16)$$

where  $\beta_j$  denotes the reliability index.<sup>(19)</sup> developed the same approach for non-normal distributions.

The reliability constraint is replaced as following deterministic constraint condition by utilizing that the design point locates on the limit state surface ( $g_i(\mathbf{d}, \mathbf{x}_{(j)}^*) = 0$ ).

$$g_j(\mathbf{d}, \mathbf{d} - \beta_j \boldsymbol{\sigma}^T \boldsymbol{\alpha}_j) \geq 0, \quad (j = 1, 2) \quad (17)$$

where the gradient vector  $\boldsymbol{\alpha}_j$  in Eq. (16) should be evaluated at the design point  $\mathbf{x}_{(j)}^*$ . However, the design point is not known a priori. That is, a conventional RBDO is formulated as a nested iteration process to find  $\mathbf{x}_{(j)}^*$ .

The key idea of the SLSV method is to use the gradient vector obtained by the previous optimization loop. The approximation makes the RBDO problem converted to a single loop. The flow of the SLSV method is described as follows.

**Step 1** Set the initial design  $\mathbf{d}^{(k)}$  and the initial sensitivity  $\boldsymbol{\alpha}_j^{(k)}$ , where  $k$  is set to 0.

**Step 2** Evaluate the limit state functions  $g_j(\mathbf{d}^{(k)}, \mathbf{x})$ , where random vector  $\mathbf{x}$  is set to  $\mathbf{d} -$

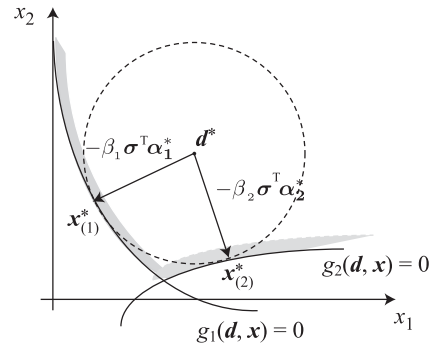


Fig. 2 Design point in  $X$ -space

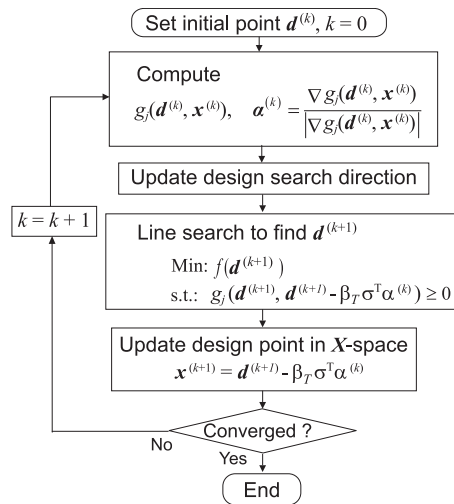


Fig. 3 Flowchart of SLSV method

$\beta_{T_j} \sigma_j \alpha_j^{(k)}$ . Then, normalized sensitivity vectors  $\alpha_j^{(k)}$  are obtained from the gradient vector,  $\nabla g_j(d^{(k)}, x)$ .

**Step 3** Find the design candidate  $d^{(k+1)}$  by one dimensional search of the following optimization problem.

$$\begin{aligned} \text{Minimize : } & f(d^{(k+1)}) \\ \text{subject to : } & g_j(d^{(k+1)}, d^{(k+1)}) - \beta_{T_j} \sigma_j^T \alpha_j^{(k)} \geq 0 \quad (j = 1, 2) \\ & d^L \leq d \leq d^U \end{aligned} \quad (18)$$

The random variable is updated from the one-dimensional searching result.

$$x^{(k+1)} = d^{(k+1)} - \beta_{T_j} \sigma_j^T \alpha_j^{(k)} \quad (19)$$

**Step 4** If the convergence criterion is satisfied,  $d^{(k+1)}$  is regarded as the optimum solution. Otherwise, set  $k$  is  $k + 1$  and go back to Step 2. for continuing the next one-dimensional search.

A flowchart of the SLSV method is illustrated in Fig. 3.

The design variables are updated using the convex linearization method<sup>(20)</sup>, a type of sequential convex programming, which is adopted for numerical optimization. This approach offers a distinct convergence advantage for stiffness problems<sup>(21)</sup>.

## 5. Numerical examples

Several numerical examples are presented to examine and verify the SLSV method in comparison with conventional double-loop approach. In the following examples, the assumed material is steel with a Young's modulus, Poisson's ratio, and mass density of 206GPa, 0.3, and 7784 kg/m<sup>3</sup>, respectively. These material constants are assumed to have deterministic values. Because the discussed problem is based on linear structural mechanics, uncertainty

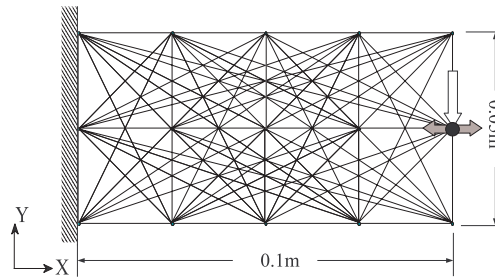
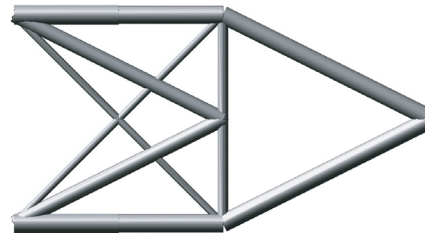


Fig. 4 Ground structure of 2D frame.


Fig. 5 Optimum configuration under reliability condition by SLSV method ( $\beta_{T_j} = 3.0$ )

of material property has a small effect on the reliability-based optimal configuration. The penalization parameter  $p$  is set to 1,0 and the allowable maximum cross-sectional diameter of each frame element is set to 0.01 m. For simplicity, all of random variables are assumed to be normally distributed in all the examples.

### 5.1. 2-D frame design

An optimization feature of the SLSV method is compared with that of the double-loop approach is compared for a two-dimensional frame structure considering both compliance and eigenfrequency criteria.

A ground structural model of this example is shown in Fig. 4. The ground structure consists of frame elements connecting any two nodes that are arranged at four equally divided locations in the  $X$  direction and two equally divided locations in  $Y$  direction. A rectangular design domain is supported at the left-hand side edge, uncertain loads ( $\mu_x = 0\text{N}$ ,  $\sigma_x = 2\text{N}$ ,  $\mu_y = -10\text{N}$ ,  $\sigma_y = 1\text{N}$ ) are applied in the  $X$  and  $-Y$  direction at the center of the right-hand side, respectively, and uncertain nonstructural mass ( $\mu = 1\text{kg}$ ,  $\sigma = 0.2\text{kg}$ ) is located at the same node.

For the limit state functions, the upper limit of the mean compliance is set to  $l^U = 2.497 \times 10^{-5}\text{J}$  and the lower limit of the mean eigenfrequency is set to  $\Lambda^L = 16.14\text{Hz}$ , where these values are determined from deterministic optimum designs, the compliance minimization and the eigenfrequency maximization under the volume constraint of 1% of the total volume. The reliability-based design is obtained for the target reliability index value of  $\beta_{T_j} = 3.0$  for the both criteria. The initial design is selected that all of possible elements have equal diameter for 3% of the total volume, that is three times of the deterministic design.

The optimum configuration obtained by the SLSV method is illustrated in Fig. 5, which is equivalent to that obtained by the conventional double loop method.

The volume history of the both method is compared in Fig. 6 (a) and the reliability indices are compared in Figs. 6 (b) and (c). Where,  $\beta_l$  and  $\beta_\Lambda$  in Figs. 6 (b) and (c) are reliability indices of the mean compliance and the mean eigenfrequency criteria, respectively. The structural volume is increased in early stage, because the initial design does not satisfy reliability constraints. Once the reliability constraints are satisfied, the structural volume is reduced less than that of the initial design with satisfying the constraints until reaching the convergence. The number of iterations until convergence is almost the same level, though the SLSV method reaches the optimum a little faster. Note that the SLSV method does not evaluate the reliabil-

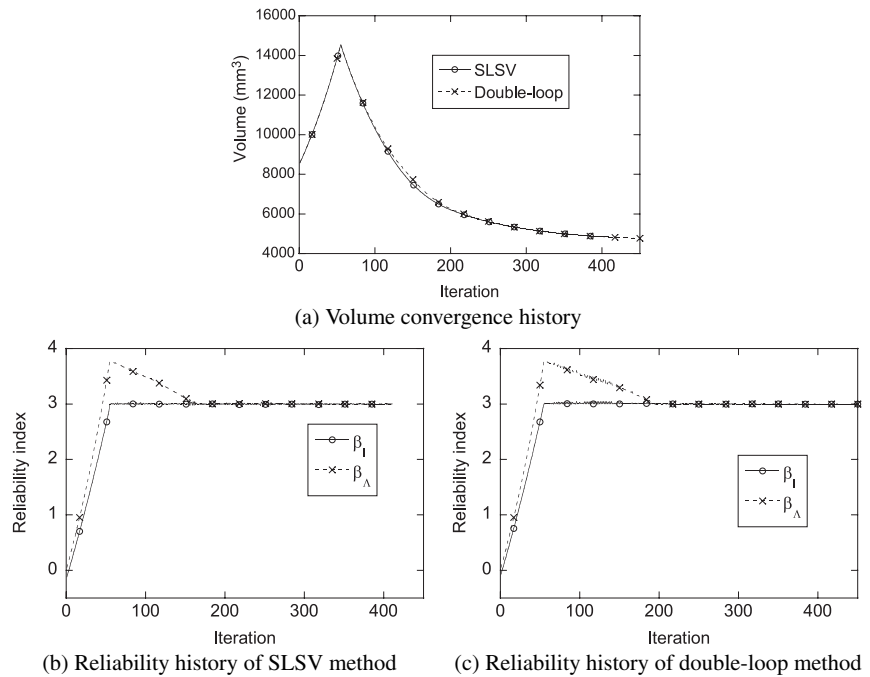


Fig. 6 Comparison of convergence histories (2D Frame)

Table 1 Comparison of computational performance (2D frame,  $\beta_{T_j} = 3.0$ ).

|                           | SLSV  | Double-loop |
|---------------------------|-------|-------------|
| $\beta_I$                 | 3.004 | 3.000       |
| $\beta_\Lambda$           | 3.003 | 2.998       |
| Volume (mm <sup>3</sup> ) | 4832  | 4769        |
| Iteration                 | 412   | 450         |
| NFE ( $g_1$ ) *           | 412   | 2966        |
| NFE ( $g_2$ ) *           | 412   | 3993        |

\* Number of function evaluation.

ity in each iteration. The value of reliability index of each iteration for the SLSV method in Fig. 6 (b) is obtained by FORM for confirmation of the reliability approximation accuracy. In both methods, the structural volume is increased until satisfying the reliability constraints in early stage, and then, the structural volume is decreased on keeping the reliability level until convergence.

The optimization results are summarized in Table 1. The reliability indices for optimum designs for both methods are evaluated by Rackwitz-Fiessler method. The results show that the SLSV method has sufficient reliability approximation accuracy in comparison with FORM. On the other hand, the evaluation number of the limit state function that includes structural analysis is less than one-eighth by the conventional method. This result indicates that adopting the SLSV for RBTO problem for frame structure is suitable for achieving computational efficiency.

## 5.2. Effect of normalization of limit state function

Convergence history for the same design problem without normalization of the limit state functions is shown in Fig. 7. The figure shows only the first 450 iterations, but the optimization has not converged over 1200 iterations. Both the objective function and the reliability indices keep on oscillating. Moreover, the reliability index of the mean compliance  $\beta_I$  is around 2.5 and not approach the target reliability  $\beta_{T_j} = 3.0$ . Nevertheless, the volume is not decreased to the normalization level shown in Fig. 6.

In this example, the allowable values of the mean compliance and the mean eigenfrequency are much different orders. Therefore, the SLSV method is considered not to be con-

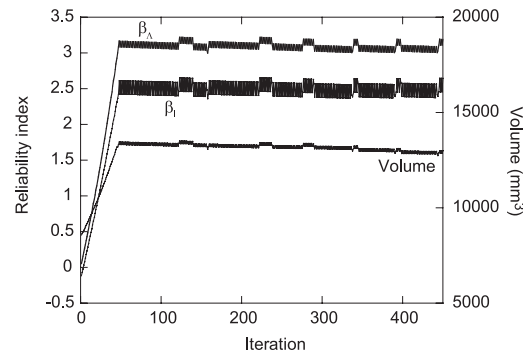


Fig. 7 Iteration history by SLSV method without normalization of limit state functions.

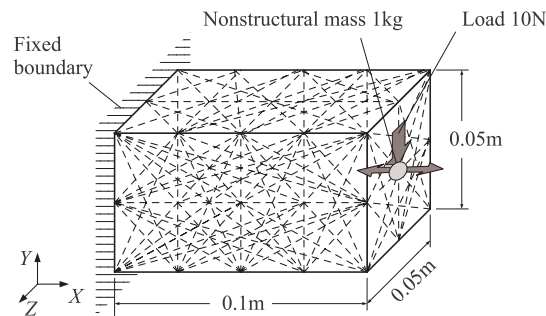


Fig. 8 Ground structure of 3D frame.

verged to the optimal solution. This result shows that the normalization of the limit state functions is important like a deterministic optimization problem, because the SLSV directly appears the limit state function in the optimization problem.

### 5.3. 3-D frame design problem

The same approach is applied to a three-dimensional frame design problem. The design domain is configured as a rectangular solid, as shown in Fig. 8. The ground structure consists of frame elements connecting any two nodes that are arranged at four equally divided locations in the  $X$  direction and two equally divided locations in  $Y$  direction, though not all of the frame elements are drawn in Fig. 8.

A structure is supported at the left-hand side edge, uncertain loads ( $\mu_x = 0\text{N}$ ,  $\sigma_x = 2\text{N}$ ,  $\mu_y = -10\text{N}$ ,  $\sigma_y = 1\text{N}$ ) is applied in the  $X$  and  $-Y$  direction at the center of the right-hand side, respectively, and uncertain nonstructural mass ( $\mu = 1\text{kg}$ ,  $\sigma = 0.2\text{kg}$ ) is located at the same node.

For the limit state functions, the upper limit of the mean compliance is set to  $l^U = 2.085 \times 10^{-6}\text{J}$  and the lower limit of the mean eigenfrequency is set to  $\Lambda^L = 36.05\text{Hz}$ , where these values are determined from deterministic optimum designs, the compliance minimization and the eigenfrequency maximization under the volume constraint of 1% of the total volume. The reliability-based designs are obtained for the target reliability index value of  $\beta_{T_j} = 3.0, 4.0, 5.0$  and  $6.0$  for the both criteria. The initial design is selected that all of possible elements have equal diameter for 3% of the total volume.

The obtained optimum configurations by the SLSV method and the conventional double-loop method are illustrated in Figs. 9 and 10, respectively. Though optimum solutions by both methods have small differences, they can be regarded as the same configurations.

Convergence history of the SLSV method is quite similar to that of the double-loop method. That in case of  $\beta_{T_j} = 3.0$  is shown in Fig. 11. In this case, the number of iteration in the SLSV method is slightly larger than that of the double-loop method. It is found that the SLSV method with normalized limit state function has sufficient convergence property. Note that the reliability index history in the SLSV method is obtained by the FORM for

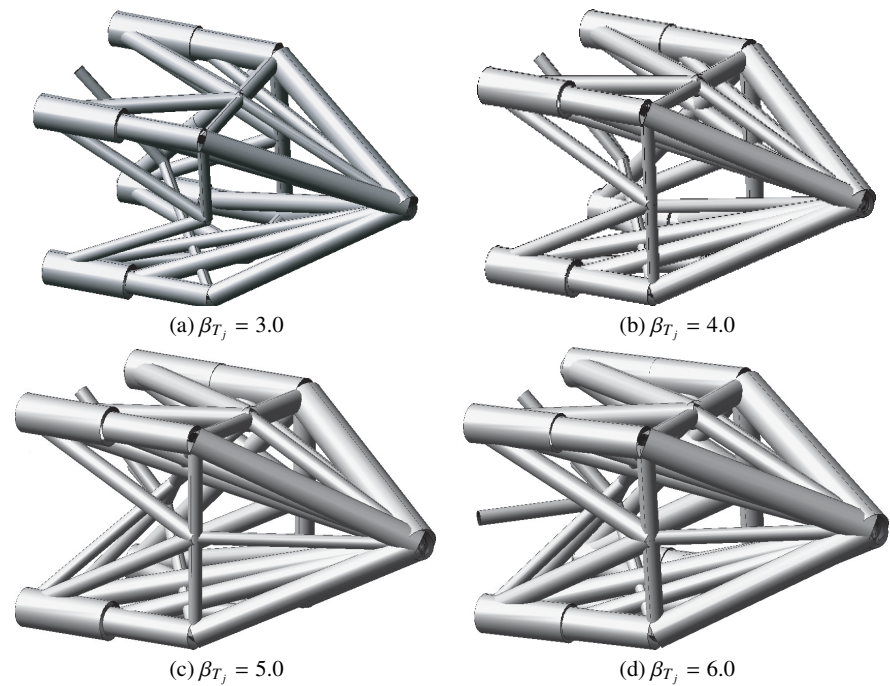


Fig. 9 Optimum configurations by SLSV method

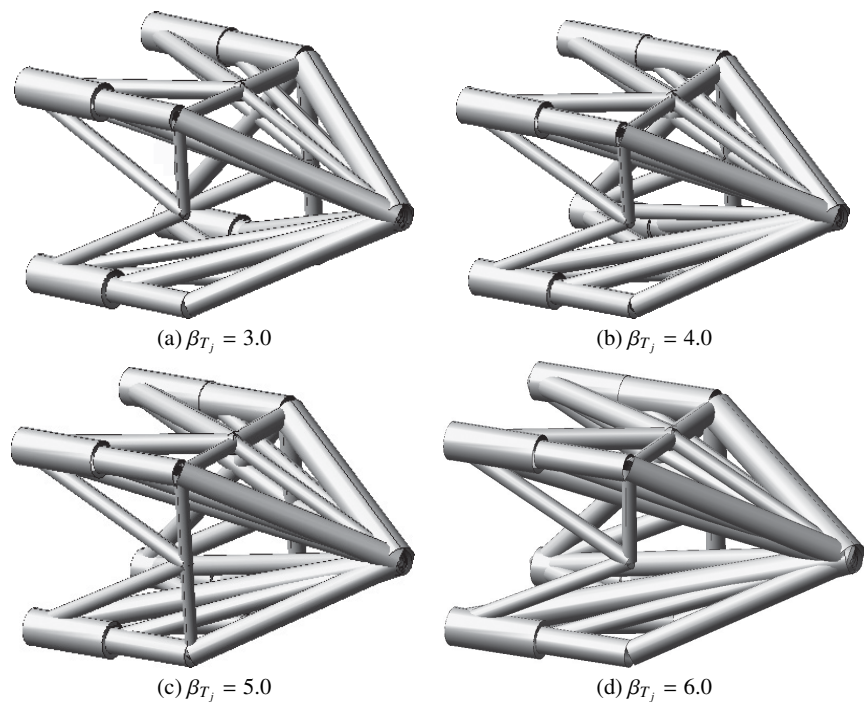


Fig. 10 Optimum configurations by double-loop method

each iteration, as described in the previous example. The similar convergence properties are obtained for the other target reliability criteria, as shown in Fig. 12, where only the volume convergence histories are shown.

The SLSV method obtains slightly smaller volume designs with the double-loop method for all target reliability criteria as listed in Table 2; 0.3% for  $\beta_{T_j} = 4.0$ , 1.4% for  $\beta_{T_j} = 4.0$ , 2.2% for  $\beta_{T_j} = 5.0$ , and 3.2% for  $\beta_{T_j} = 6.0$ . In addition, the difference between the converged reliability and the target reliability is slightly larger for the SLSV method as shown in Table 2. However, the reliability approximation accuracy is sufficient for all the cases. Note that the optimum designs by the double-loop method slightly violate the target reliability criteria for

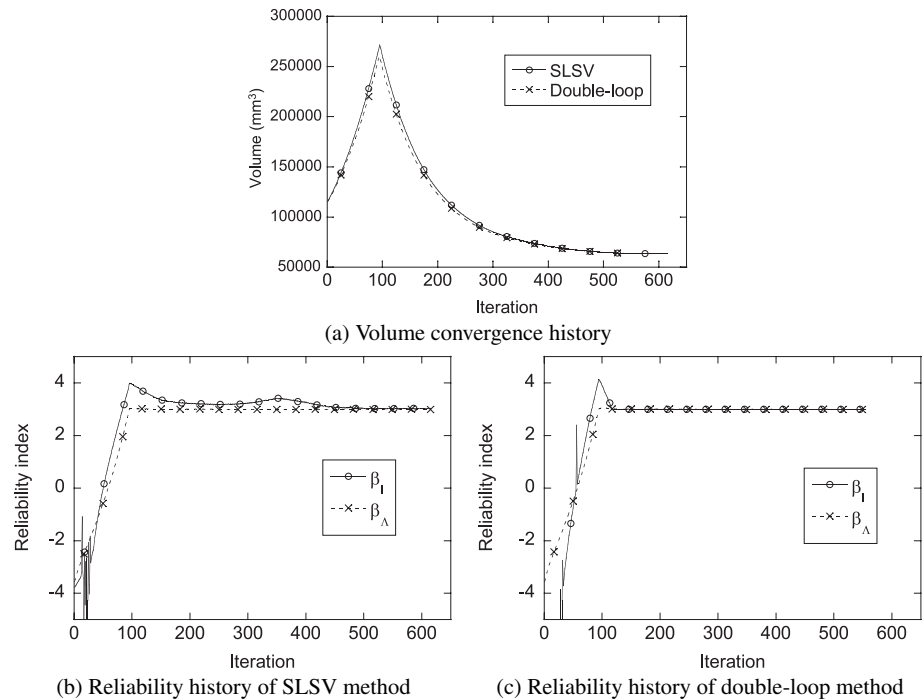


Fig. 11 Comparison of convergence histories ( $\beta_{T_j} = 3.0$ )

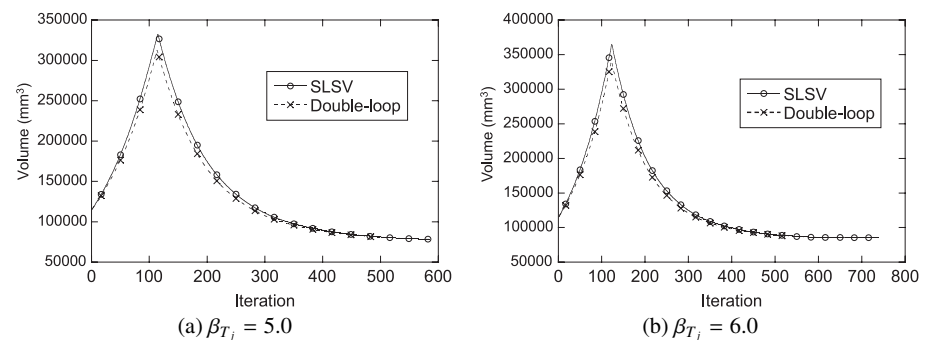


Fig. 12 Comparison of volume convergence history

the eigenfrequency criterion. In addition, the optimum configurations for both methods are almost identical to both methods described above. Therefore, the SLSV method is sufficient accuracy for the reliability approximation in this frame structural design problem.

Calculation efficient of the SLSV method is also compared with that of the double-loop method in Table 2. It is found that the computational efficiency of the SLSV method is quite attractive. The number of limit state function calculation with structural analysis is at least 10 times less than that of the double-loop method. It indicates that the SLSV method with limit state function normalization is possible to apply for a large-scale problem.

## 6. Conclusion

This paper applies SLSV (Single-Loop-Single-Vector) method to reliability-based topology optimization for frame structures and investigates the reliability accuracy and computational efficiency of SLSV method. From numerical examples, it is shown the computational efficiency of the RBTO for frame structure with multiple criteria.

In addition, the normalization of the limit state functions is introduced for achieving stable numerical searching in the optimization. Such a normalization is a common knowledge of deterministic optimization. However, in the conventional double-loop method, limit state functions do not appear in the optimization loop explicitly, but only in the reliability analysis loop such as FORM. Since the reliability is evaluated in each mode separately, it is not

Table 2 Comparison of computational performance of 3D frame.

| (a) SLSV method           |       |       |       |       |
|---------------------------|-------|-------|-------|-------|
| $\beta_{T_i}$             | 3.0   | 4.0   | 5.0   | 6.0   |
| $\beta_l$                 | 3.024 | 4.000 | 5.077 | 6.000 |
| (Accuracy)                | +0.8% | 0%    | +1.5% | 0%    |
| $\beta_\Lambda$           | 2.988 | 3.996 | 4.992 | 5.990 |
| (Accuracy)                | -0.4% | -0.1% | -0.2% | -0.2% |
| Volume (mm <sup>3</sup> ) | 63581 | 71143 | 78544 | 85547 |
| Iteration                 | 615   | 554   | 585   | 739   |
| NFE ( $g_1$ ) *           | 616   | 555   | 586   | 740   |
| NFE ( $g_2$ ) *           | 616   | 555   | 586   | 740   |

| (b) Double-Loop method    |       |       |       |        |
|---------------------------|-------|-------|-------|--------|
| $\beta_{T_i}$             | 3.0   | 4.0   | 5.0   | 6.0    |
| $\beta_l$                 | 3.000 | 4.000 | 5.000 | 6.001  |
| (Accuracy)                | 0%    | 0%    | 0%    | +0.01% |
| $\beta_\Lambda$           | 2.997 | 3.991 | 4.994 | 5.994  |
| (Accuracy)                | -0.1% | -0.2% | +0.1% | -0.1%  |
| Volume (mm <sup>3</sup> ) | 63791 | 72150 | 80318 | 88370  |
| Iteration                 | 555   | 517   | 512   | 516    |
| NFE ( $g_1$ ) *           | 6358  | 7349  | 10769 | 13565  |
| NFE ( $g_2$ ) *           | 10075 | 9307  | 9492  | 9629   |

\* Number of function evaluation.

necessary to take care of the order of limit state functions for mode reliability problem and even system reliability problem. On the other hand, SLSV method explicitly uses limit state functions in optimization loop as constraints. Through numerical example, the importance of the normalization of the limit state functions is demonstrated.

### Acknowledgements

This work was supported by the International Cooperation Program funded by KOSEF (F01-2005-000-10252-0). In addition, the author would like to express our acknowledgements to Dr. Akihiro Takezawa, Hiroshima University, Japan for his valuable comments.

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