

Control of the Half-Skyrmion Hall Effect and Its Application to Adder–Subtractor

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Skyrmions have attracted a great attention in spintronics because of their potential use as robust information carriers with distinctive protection. Though the realization of skyrmion-based devices requires flexible control of a skyrmion motion, achieving such a skyrmion motion has been hampered by the skyrmion Hall effect (SkHE), which refers to the presence of a finite angle between a current and the skyrmion trajectory. Here, new insight for the precise control of half-skyrmion motion is presented, including complete suppression of the SkHE by deforming the internal structure of skyrmions, which is experimentally achieved by external magnetic field to steer current-driven half-skyrmions in the desired direction. Furthermore, based on the unique advantages in half-skyrmions, the potential of half-skyrmions application beyond skyrmion-based electronics is also demonstrated by presenting simple half-skyrmion-based addition/subtraction operation. The findings of controllability of 2D half-skyrmion motion will provide new perspectives on utilization of topological solitons for device applications.

1. Introduction

Since the first observation of magnetic skyrmion,^[1] a variety of works on the skyrmion show diverse novel topological phenomena including emergent electromagnetism,^[2,3] topological Hall effect,^[4] and skyrmion Hall effect (SkHE).^[5,6] In particular, their topologically protected particle-like behavior, along with outstanding efficiency of current-driven motions enable magnetic skyrmions to become one of the most promising alternatives serving as robust information carriers in the next-generation spintronic devices.^[7] In this regard, attaining a firm understanding of the current-driven skyrmion dynamics remains a key step towards extending their use in spintronic applications such as skyrmion-based memories,^[8,9] skyrmion neuromorphic devices,^[10] and

skyrmion logic devices.^[11] Considering magnetic skyrmions as a rigid body, the current-driven skyrmion dynamics are well described by the Thiele's approach under the current as follows^[3,12]

$$\mathbf{G} \times \mathbf{v} - \alpha \vec{D} \cdot \mathbf{v} + \mathbf{F} = 0 \quad (1)$$

where the skyrmion motions are in the xy -plane, $\mathbf{G} = -4\pi Q \hat{z}$ is the gyromagnetic coupling vector with the topological number Q , which measures how many times the normalized magnetization vector $\mathbf{m}$ wraps the unit sphere, defined as $Q = \frac{1}{4\pi} \int \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m}) dx dy$; $\mathbf{v}$ is the drift velocity of skyrmions, α is the damping constant, $\vec{D}$ is the dissipation tensor and $\mathbf{F}$ is the driving force provided by the current. The first term of Equation (1) implies the inherent presence of transverse components in drift velocity with respect to the $\mathbf{F}$ direction, thereby causing the deflected motion of the current-driven skyrmions (called as “SkHE”). As a result, the skyrmion meets the edges of patterned devices^[13] due to the SkHE. If the repulsive force generated at the edge is large enough, the skyrmion will move stably along the edge. However, if current is too high to push the skyrmion towards the edge, the skyrmion can annihilate. Thus, the widespread use of skyrmions in spintronic applications significantly needs precise manipulation of SkHE, along with unlocking their underlying physics. For this issue, there have been several suggestions to eliminate the SkHE by using antiferromagnets or synthetic antiferromagnets,^[13,14] where the net topological number of two coupled skyrmions

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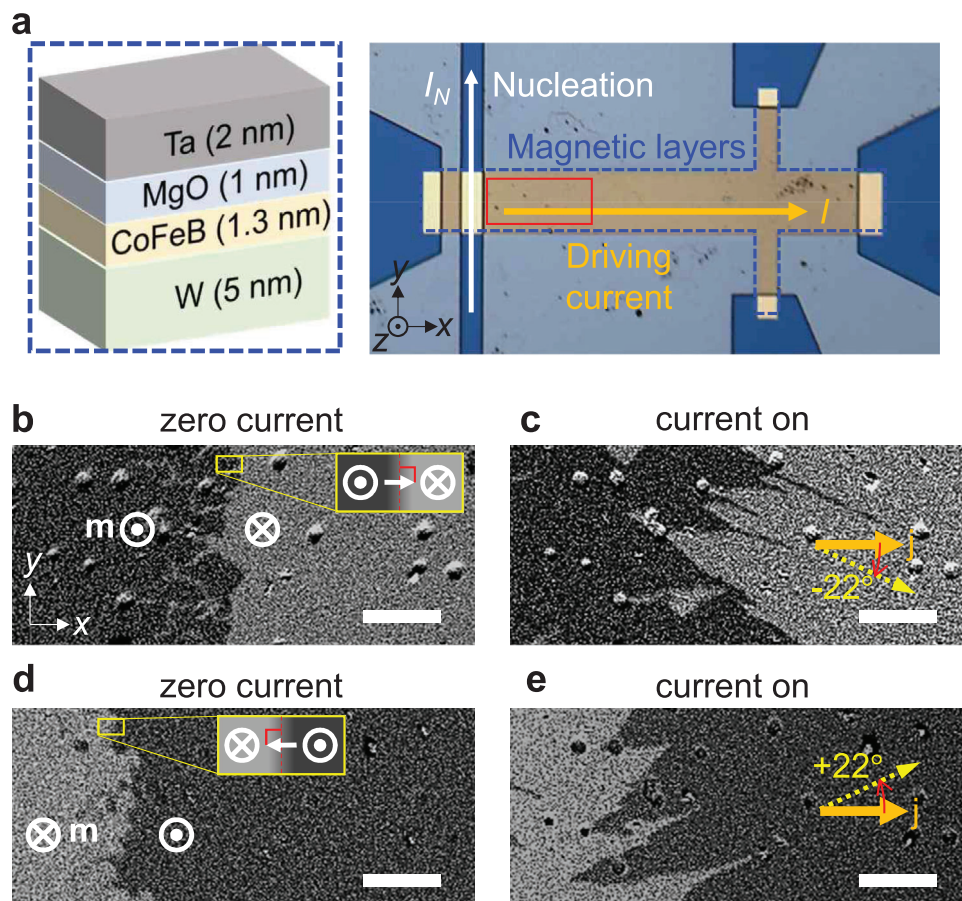


Figure 1. Chiral stripe domain elongation with different core magnetization configurations. a) Schematic diagram of the W/CoFeB/MgO/Ta multilayer stack (left) and optical micrograph of the fabricated microwire. The red boxed area is observed. b) Initial domain formation after nucleation with the nucleation current $I_N = +0.11$ A. Dark (light) grey corresponds to $+z$ ($-z$) magnetization domains. c) Up-stripe domain elongation effect by the x -axis current injection with the current density, $j = +6.38 \times 10^6$ A cm $^{-2}$, which has an angle of -22° with respect to the x -axis (current direction). d) Initial domain formation with the reversed magnetization configuration through the opposite nucleation current $I_N = -0.11$ A. e) Down-stripe domain elongation by the same current ($j = +6.38 \times 10^6$ A cm $^{-2}$) with the opposite angle, $+22^\circ$. All scale bars are 10 μ m.

with opposite charges is zero. However, these approaches can only serve the zero topological number so that fine manipulation of the SkHE cannot be achieved. One possible approach to control the SkHE accurately has very recently been suggested,^[15] that is, controlling of the angular momentum in ferrimagnets with variations of the temperature. However, a suitable way to suppress and even exactly control the SkHE for integration with complementary metal-oxide-semiconductor (CMOS) devices is still lacking. By addressing this open issue, we present that the internal structure of skyrmions can be a key for manipulating the SkHE. In this work, the half-SkHE is substantially modulated even in its sign by deforming the internal structure of half-skyrmion, which was achieved by employing external magnetic fields. It demonstrates that the internal structure is a novel approach to control the SkHE beyond material parameter control or topological number engineering. In addition, it is worth noting that the half-skyrmions present its unique advantages which cannot be realized by full-skyrmions. The half-skyrmions leave a trace along its trajectory during moving. This unique characteristic can make half-skyrmions distinct from full-skyrmions. For example, a recent paper^[16] reported the imaging of inhomoge-

neous current path using the remained trajectory half-skyrmion motion, which cannot be realized by full-skyrmion. Especially, for a certain logic application, where more than two skyrmions should interact with each other, this characteristic can be more efficient when adopting half-skyrmions. For interacting two full-skyrmions, the arrival time of the full-skyrmions at the particular position should be synchronized, which is technically hard to achieve. However, half-skyrmions do not need to be time synchronized because they leave a trace. Therefore, we believe that half-skyrmions are another significant research subject for investigating novel magnetic phenomena (related to topological physics) and also for novel spintronics application beyond full-skyrmion-based application.

2. Results and Discussion

To demonstrate the dependence of half-skyrmion dynamics on the external magnetic field, we investigated Si/SiO $_2$ /5 nm W/1.3 nm Co $_{20}$ Fe $_{60}$ B $_{20}$ /1 nm MgO/3 nm Ta films. Each sample was patterned into a typical Hall bar shape, as shown in Figure 1a. The films have the perpendicular magnetic anisotropy

(PMA) that prefers $+z$ or $-z$ directions of magnetization. After magnetic saturation of the entire film along the $+z$ direction using the $+z$ external magnetic field, we injected a current pulse (0.11 A) through the nucleation line (Figure 1a) that generates the Oersted field, where the Oersted field is around 70 Oe, which is higher than the coercivity field (Supporting Information S2). The Oersted field reverses the magnetization near the nucleation line to the $-z$ direction. The magnetic domains were investigated using a magneto-optical Kerr-effect microscope (MOKE). The $+z$ ($-z$) magnetization domain is shown as dark (light) grey in Figure 1b. Then, at the transition region between the two different domains, a domain wall with in-plane magnetization components is formed for lowering the exchange energy.^[17] Because the studied film exhibits the finite Dzyaloshinskii–Moriya interaction (DMI) constant of $\approx -0.43 \text{ mJ m}^{-2}$, confirmed by Brillouin light scattering measurement (see Supporting Information S1), the domain walls are expected to have the chiral Néel-type configuration (insets of Figure 1b,d). In this configuration, as the result of spin–orbit torques (SOT) on the chiral domain walls, the electrical current along the x -axis (orange arrow in Figure 1) in the W layer induces the stripe domain elongation as confirmed in Figure 1c,e. According to a recent study^[15] on the elongation of a pinned magnetic bubble driven by SOT, the overall dynamics is determined by the motion of a half-skyrmion with topological charges $Q = \pm 1/2$ at the end of the stripe domain.^[18] Thus, examining the elongation angle of the stripe domain corresponds to investigating the SkHE of a half-skyrmion.

The limitation of the current-induced elongation of the stripe domains in the absence of an external magnetic field can be understood as follows. When a current is applied to the up-stripe domain, it elongates with an angle of -22° measured counter-clockwise from the current direction (Figure 1c). However, the opposite domain (down)-stripe elongation has an opposite angle, $+22^\circ$. The result indicates that the magnetization direction of the stripe domain correlates with the sign of the stripe domain elongation angle. This sign reversal of the angle with the inversion of magnetization configurations can also be explained by the SkHE of a half-skyrmion with the following relation obtained from the above Thiele’s equation for the SkHE of a half-skyrmion:^[15]

$$\theta_{\text{SkHE}} \approx \tan^{-1} \left(\frac{2Q\lambda}{\alpha r} \right) \quad (2)$$

where θ_{SkHE} is the skyrmion Hall angle (SkHA), λ is the domain wall width and r is the radius of the half-skyrmion. Based on the above theory accounting for the current only, the manipulation of the SkHA is possible only through variations of α , r , and λ .^[8,15,19] However, among these parameters, the radius of the half-skyrmion determines the device size because the half-skyrmion is an information carrier. Thus, it can be a challenging issue to change α , λ with keeping the half-skyrmion size. Therefore, for a new method for SkHE manipulation, we generalize the existing theory for a chiral stripe domain elongation to the cases with the simultaneous presence of external magnetic fields and SOT.

To understand the skyrmion (including half-skyrmion) motion subjected to the external magnetic field, the governing equation of motion for skyrmions, Thiele’s equation, needs to be thoroughly investigated. Thiele’s approach starts with the assumption that the non-collinear spin texture moves maintaining

its configuration, that is, $\dot{\mathbf{m}} = -(\dot{\mathbf{R}}_{\text{Sk}} \cdot \nabla) \mathbf{m}_0$, where $\mathbf{R}_{\text{Sk}}$ describes the position and $\mathbf{m}_0$ is the magnetization configuration. Plugging the above condition into the Landau–Lifshitz–Gilbert (LLG) equation under the magnetic field and the damping-like SOT, LLG becomes

$$-(\dot{\mathbf{R}}_{\text{Sk}} \cdot \nabla) \mathbf{m}_0 = -\gamma_0 \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha \mathbf{m} \times (\dot{\mathbf{R}}_{\text{Sk}} \cdot \nabla) \mathbf{m}_0 + \gamma_0 \tau_d \mathbf{m} \times (\boldsymbol{\sigma} \times \mathbf{m}) \quad (3)$$

Here, γ_0 is the gyromagnetic ratio, $\mathbf{H}_{\text{eff}} (= \mathbf{H}_{\text{int}} + \mathbf{H}_{\text{ext}})$ is the total effective magnetic field, $\mathbf{H}_{\text{int}}$ is the internal magnetic field, $\mathbf{H}_{\text{ext}}$ is the external magnetic field, τ_d is the strength of damping-like SOT, and $\boldsymbol{\sigma}$ is the pumped spin direction. The main idea of Thiele’s approach is to convert this equation of motion into a transitional equation of motion by multiplying $\mathbf{m}_0 \times \partial_i \mathbf{m}_0$ ($i = x, y$) and integrating the resultant equation over the magnetic configuration. Then, the consequent Thiele’s equation for the motion of skyrmions with $\mathbf{H}_{\text{ext}} = 0$ is given as Equation (1) because $\mathbf{H}_{\text{int}}$ is parallel to $\mathbf{m}_0$. If the external magnetic fields are added, an additional reduced force is generated as $\int (-\gamma_0 \mathbf{m} \times \mathbf{H}_{\text{ext}}) \cdot (\mathbf{m}_0 \times \partial_i \mathbf{m}_0) dx dy$. The word “reduced” is used here because the considered forces are normalized by the areal angular momentum density ($\text{J} \cdot \text{s m}^{-2}$) and thus have units of velocity (m s^{-1}). The forces discussed hereafter should be understood as the reduced forces. The generation of an additional force in Thiele’s equation by an external magnetic field indicates that the skyrmion motion can be modulated by it.

2.1. Chiral Stripe Domain Elongation under Out-of-Plane Magnetic Fields

Hirata et al.^[15] explained the elongation of chiral magnetic bubbles by considering a balance between several types of forces exerted on the rigid half-skyrmion end: the gyrotropic force $\mathbf{F}_g = -4\pi Q v (\hat{\mathbf{z}} \times \hat{\mathbf{v}})$, the pinning force $\mathbf{F}_p = -F_p \hat{\mathbf{v}}$, the dissipative force $\mathbf{F}_d = -\alpha D v \hat{\mathbf{v}}$ and the SOT force $\mathbf{F}_j = \frac{\pi^2 \gamma_0 \tau_d r}{2} \hat{\mathbf{x}}$, where v is the half-skyrmion speed, $\hat{\mathbf{v}}$ is the unit vector of the elongation direction, and D is the strength of the dissipative tensor. Note that the analytical results presented below hold only for half-skyrmions and the SkHE represents the Hall effect of a half-skyrmion. As explained above, the stripe elongation angle in Figure 2a corresponds to the SkHA (θ_{SkHE}) of the half-skyrmion. Then, if the out-of-plane external field ($H_z \hat{\mathbf{z}}$, the field strength is H_z) is switched on, an additional force is created (Figure 2a). The total force from $H_z (\mathbf{F}_{H_z})$ on the rigid half-skyrmion can be obtained by integrating the local force (f_{H_z} in Figure 2b), which pushes the domain wall in the normal direction (red arrows in Figure 2b), over the entire domain wall region. The resultant force is given by $F_{H_z} = 4\gamma_0 H_z r \hat{\mathbf{v}}$ (the detailed derivation is presented in Supporting Information S3), showing that H_z always induces an additional force along the $\hat{\mathbf{v}}$ direction. By balancing the force components along the $\hat{\mathbf{v}}$ direction as well as the $\hat{\mathbf{z}} \times \hat{\mathbf{v}}$ direction respectively, and removing the v , the relation between θ and H_z can be obtained as

$$\frac{\pi^2 \gamma_0 \tau_d r}{2} \cos \theta + \frac{\alpha D}{4\pi Q} \frac{\pi^2 \gamma_0 \tau_d r}{2} \sin \theta - F_p + 4\gamma_0 H_z r p = 0 \quad (4)$$

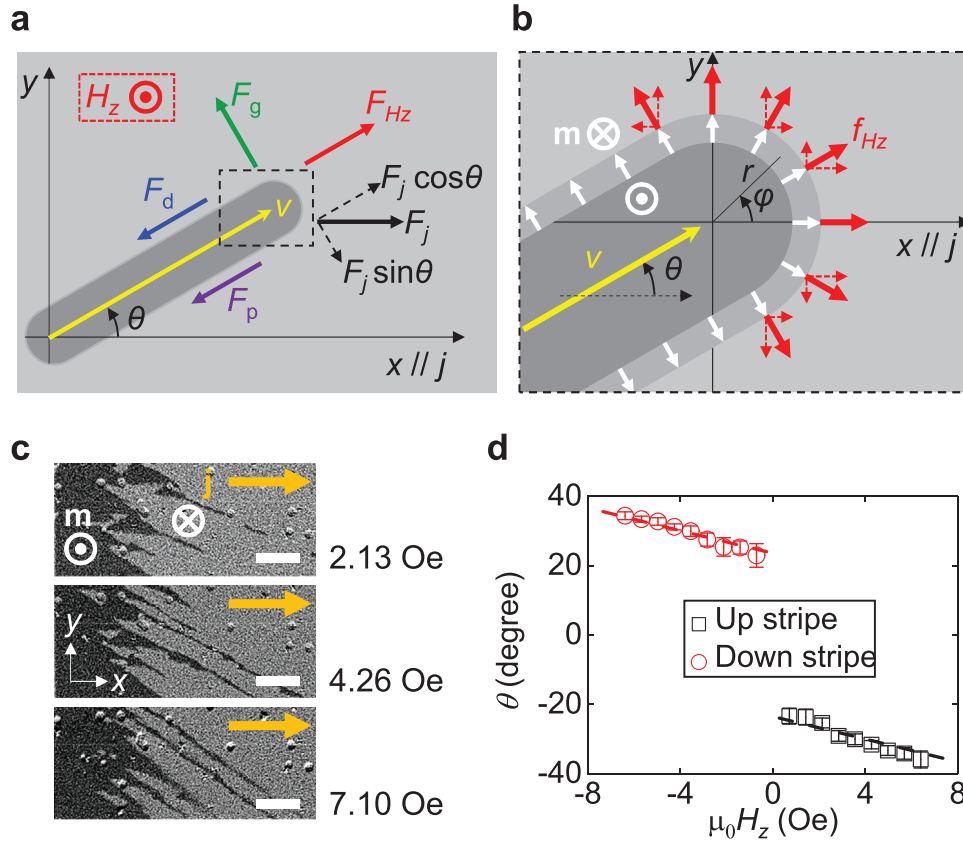


Figure 2. Dependence of the out-of-plane external magnetic fields on the chiral stripe domain elongation angle. a) Schematic illustration of the chiral stripe domain elongation and various forces on the stripe domain. F_{Hz} is created by the out-of-plane magnetic field. b) Enlarged figure of the black dashed area in (a). Local forces on each magnetization in the domain walls by the out-of-plane magnetic field. c) Representative chiral stripe domain elongation as a function of the magnitude of the out-of-plane magnetic field of 2.13 Oe, 4.26 Oe, and 7.10 Oe under the x -axis current injection of $j = +6.38 \times 10^6 \text{ A cm}^{-2}$. The scale bars are 10 μm . d) Overall out-of-plane magnetic field dependence on the chiral stripe domain elongation angle (θ) for the up-stripe (black squares) and down-stripe (red circles) domains with black (red) dashed lines evaluated from Equation (4). Error bars were determined from 20 measurements.

where $p = \pm 1$ represents the magnetization polarity m_z at the half-skyrmion core. In this work, we neglect the effect of the field-induced change of half-skyrmion size by working within linear response. In addition, the strength of the dissipative tensor ($\mathcal{D}$) can be written as $\mathcal{D} = 2\pi r / \lambda^{[15]}$ (also see Supporting Information S3). Our theoretical result shows that the SkHA θ can be experimentally modulated by the out-of-plane external field even if all other parameters including r_α , Q_λ , and τ_d are fixed. It is notable that this controllability is not possible in the full-skyrmion because the integration of f_{Hz} becomes zero in the full-skyrmion.

To validate this theory, we examined the change in the stripe domain elongation angle as a function of the z -axis external field. Figure 2c presents MOKE images of the up-stripe domain elongation by SOT under three different magnitudes of H_z . The result clearly indicates that the $+z$ -axis external field acts to help the typical SkHE and consequently increases the absolute value of the stripe domain elongation angle. Figure 2d presents a summary of the stripe domain elongation angles with respect to the $\pm z$ -axis field for both up and down stripe domains. The black and red dashed lines in Figure 2d show Equation (4) as a function of H_z for the up ($Q = +1/2$) and down ($Q = -1/2$) stripe domains, respectively, with the following parameters:

$\lambda = 7.16 \text{ nm}$, $r = 545 \text{ nm}$, $\alpha = 0.03$ and $\tau_d = 9.8 \text{ Oe}$. Compared with the experimental data shown as black squares and red circles in Figure 2d, the experimental result is consistent with the above theoretical model based on half-skyrmions and demonstrates that the SkHE can be controlled by the out-of-plane external field. Equation (4) indicates that H_z competes with F_p , meaning that we can adjust the relative strength of the effective pinning ($F_p - 4\gamma_0 H_z r p$) with respect to SOT. When $4\gamma_0 H_z r p = F_p$, the effective pinning disappears and the SkHA is described by Equation (1). However, a smaller H_z causes a larger effective pinning, which reduces the SkHA. This is qualitatively consistent with the previous observation that a nonzero SkHA requires sufficient SOT to overcome the pinning effect.^[15,20]

2.2. The Chiral Stripe Domain Elongation under In-Plane Magnetic Fields

In the above case with only the out-of-plane magnetic field, the SOT force, $F_j = \frac{\pi^2 \gamma_0 \tau_d}{2} \hat{x}$, is always parallel to the current direction ($\hat{x}$) without regard to the presence of the out-of-plane magnetic field. However, the y -axis external field ($H_y \hat{y}$) can tilt

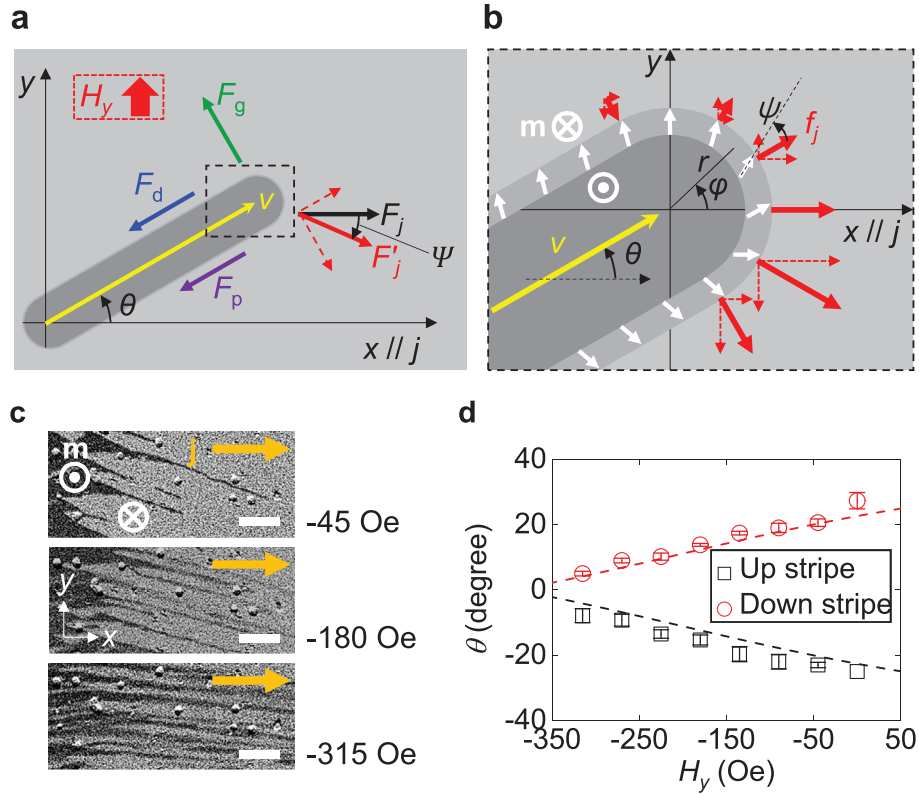


Figure 3. Dependence of the y-axis external magnetic fields (H_y) on the chiral stripe domain elongation angle. a) Schematics of the chiral stripe domain elongation-related forces on the stripe domain. F_j , the current-induced driving force, is tilted towards F_j' by H_y . b) Enlarged figure of the black dashed area in (a). Changes in local magnetization configurations and the consequent local forces by the current. c) Representative chiral stripe domain elongation as a function of the magnitude of H_y of -45 , -180 , and -315 Oe under the x-axis current injection of $j = +6.38 \times 10^6$ A cm $^{-2}$. The scale bars are 10 μ m. d) Summary of the H_y -dependent chiral stripe domain elongation angle (θ) for up-stripe (black squares) and down-stripe (red circles) domains with the fitting line of Equation (5). Error bars were obtained from 20 measurements.

the local magnetization angle of the wall by ψ at each domain wall angle (ϕ) as illustrated in **Figure 3b**. This magnetization tilting changes the reduced SOT forces exerted on the individual magnetizations in the domain wall, where the magnitudes of the reduced SOT forces at each magnetization are described as length of red arrows in **Figure 3b**. It is worth noting that the magnitude of reduced SOT force is relevant to an angle between the current direction and the magnetization direction. Thus, the total SOT-force exerted on the half-skyrmion (F_j) is also tilted by Ψ as a consequence of $\int_{\theta-\frac{\pi}{2}}^{\theta+\frac{\pi}{2}} f_j d\phi$ (the detailed derivation is in Supporting information S3).^[21] By generalizing the theory for chiral bubble elongation (Equation (2)) with the account of the effects of the tilting induced by the in-plane magnetic field, we obtain the following equation for the SkHA:

$$\frac{\pi^2 \gamma_0 \tau_d r}{2} \left[\cos \theta - \frac{4}{3\pi} \frac{H_y}{H_{DMI}} \sin \theta \cos \theta + \frac{\alpha D}{4\pi Q} \sin \theta + \frac{\alpha D}{4\pi Q} \frac{4}{3\pi} \frac{H_y}{H_{DMI}} \cos 2\theta \right] - F_p + 4\gamma_0 H_z r p \cong 0 \quad (5)$$

The result indicates that the SkHA can be controlled by both H_y and H_z . **Figure 3c** presents MOKE images for the stripe domain

elongation with three different magnitudes of H_y (indicated on the right side of the images). H_y clearly suppresses the typical SkHE. A summary of the dependence of the stripe domain elongation angle on H_y is shown in **Figure 3d**. The analytical lines (black and red dashed lines) from equation (5) match the experimental data well (black squares and red circles) for both up- and down-stripe domain elongation, where $H_{DMI} = 595.82$ Oe (obtained from the aforementioned experimental value of the DMI coefficient) is used in addition to the other parameters used for out-of-plane magnetic fields. Clearly, the SkHA can be manipulated by the external magnetic fields.

2.3. The Straight Motion of Half-Skyrmions

Let us now discuss how we can achieve the straight motion of current-driven half-skyrmions, which is an important problem in spintronics, in particular for the utilization of skyrmions in spintronic devices. According to equation (5), the condition for the straight motion ($\theta = 0$) under the external magnetic field is given by

$$\frac{\pi^2 \gamma_0 \tau_d r}{2} \left[1 + \frac{\alpha r}{2Q\lambda} \frac{4}{3\pi} \frac{H_y}{H_{DMI}} \right] - F_p + 4\gamma_0 H_z r p = 0 \quad (6)$$

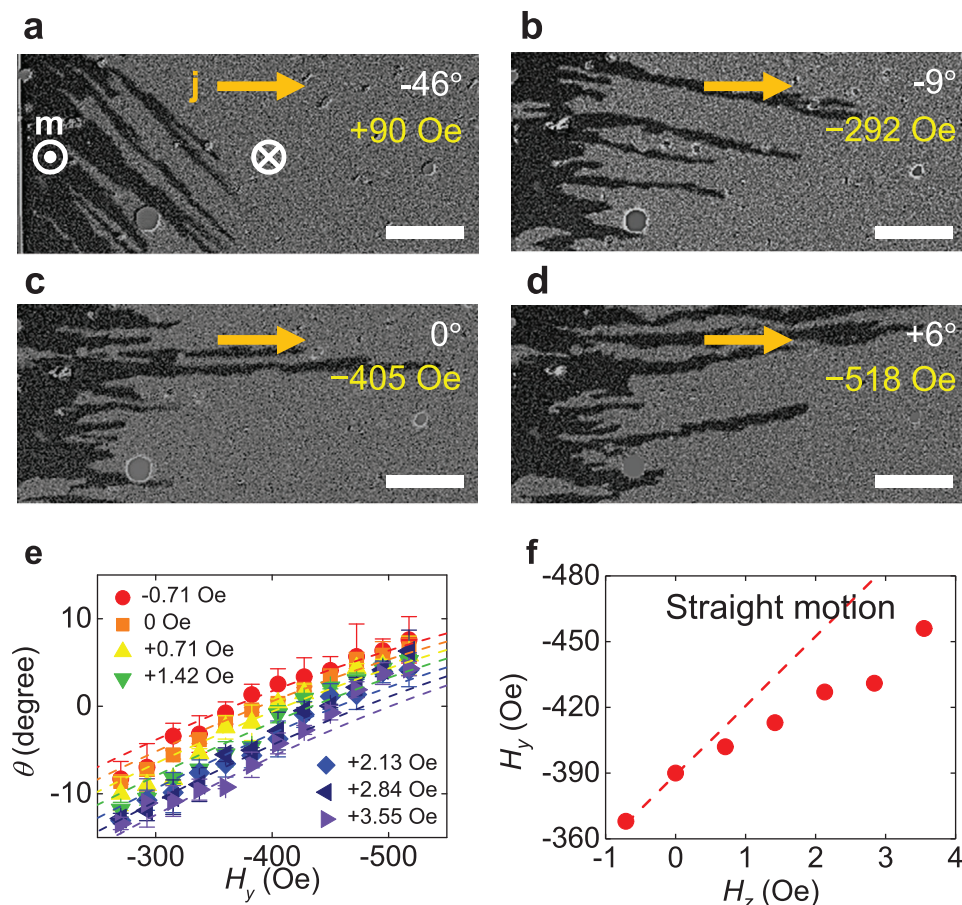


Figure 4. Straight motion of the chiral domain wall. Variation of the chiral stripe domain elongation angle under the y-axis magnetic field of a) $+90$ Oe, b) -292 Oe, c) -405 Oe, and d) -518 Oe under the x-axis current injection of $j = +6.38 \times 10^6$ A cm $^{-2}$. In (c) the chiral stripe domain is shot along the current direction ($\otimes$) under a certain condition of the external field. e) The chiral stripe domain elongation angles in the simultaneous presence of the y - and z -axis magnetic fields. Dashed lines are fitted from Equation (6). Error bars were obtained from 20 measurements. In (f), red dots represent experimentally determined y - and z -axis magnetic fields for the straight motion and the dashed line is the plot from Equation (6).

The theoretical result indicates that, under suitable choice of H_y and H_z fields, a straight motion of the half-skyrmion can be achieved. To demonstrate this approach for quenching the SkHE, the up-stripe domain elongation angles were studied under the simultaneous presence of H_y and H_z . The stripe domain elongation with fixed H_z ($= +0.71$ Oe) and different values of H_y is shown in Figure 4a–d, which evidences the change of the sign of the SkHA with respect to H_y . In particular, Figure 4c represents the straight motion of the stripe domain elongation as the result of the completely compensated SkHE of the half-skyrmion. The stripe domain elongation angle variation under diverse H_y and H_z conditions were investigated (see Figure 4e) and several combinations of the H_y and H_z for the straight motion are plotted in Figure 4f. Compared with the fitted lines (dashed lines) of Equation (6), the lines in both Figure 4e,f deviate from the experimental data in the region of high H_y . This might be due to the breakdown of certain approximations used in our theory such as the small domain-wall tilting angle $\psi \ll 1$ or due to the unaccounted change of certain skyrmion parameters such as the bubble radius r by H_z .

2.4. The Operation of Adder–Subtractor with Accumulator

In conventional CMOS systems, the four fundamental arithmetic operations ($+$, $-$, $\times$, $\div$) can be achieved by diverse combinations of full adders. However, as CMOS-based full adders are composed of two half adders and one OR logic gate for dealing with a carry operation, even simple calculations require several steps of logic processes. As a potential solution to this issue, based on the demonstrated precise control of the SkHE, we propose outstandingly simple analog addition/subtraction devices. Interesting things of these addition/subtraction devices are that the devices contain an accumulator that stores intermediate arithmetic and two different operations ($+$, $-$) can be achieved in a single device, which is not possible in conventional CMOS circuits. Figure 5a–c indicate three basic building blocks of addition operations, which corresponds to the “ -1 ,” “ $+0$,” and “ $+1$ ” operations, respectively. Each operation is activated by a current pulse of 6.38×10^6 A cm $^{-2}$ with length 50 m along the x -axis under H_z of -0.75 , 0.75 , and 1.5 Oe and H_y of -518 , -292 , and $+90$ Oe, respectively. The nucleation occurs at naturally created

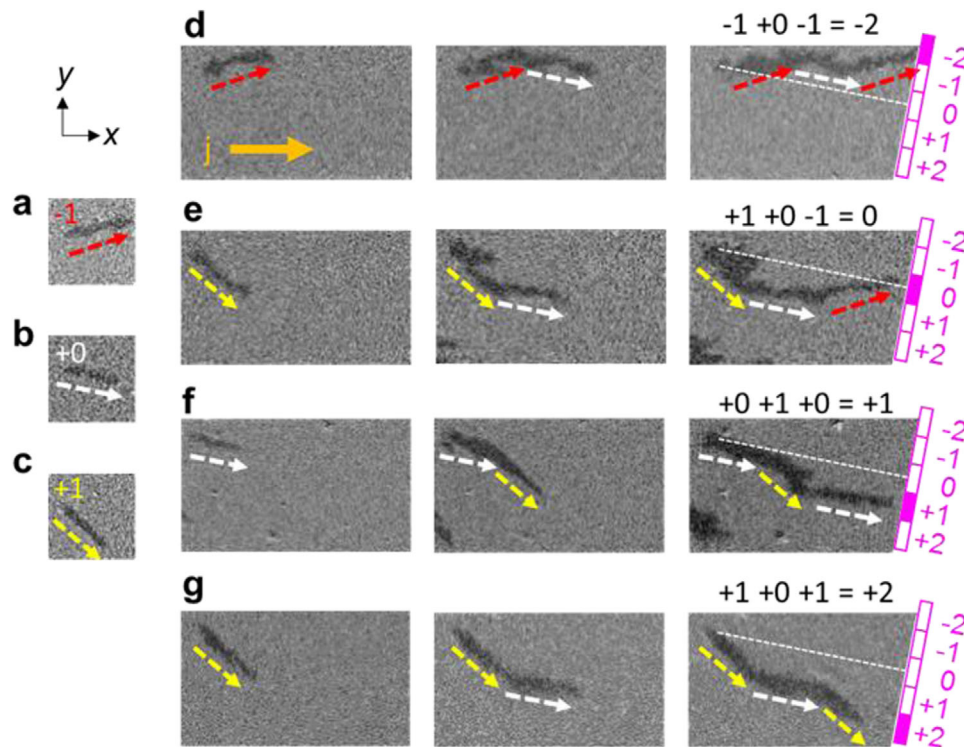


Figure 5. Operation of addition by the control of the chiral stripe domain elongation. Basic block operations for the Addition operations as a) “−1,” b) “+0” and c) “+1.” Each operation is carried out under certain combinations of H_z and H_y by the x-axis current pulse of 50 ms with the current density of $j = +6.38 \times 10^6 \text{ A cm}^{-2}$. The specific external magnetic field sets (H_y , H_z) are (−518 Oe, −0.71 Oe), (−292 Oe, +0.75 Oe), and (+90 Oe, +1.42 Oe), respectively. Four examples of the addition operations are presented. d) “−1,” “+0,” “−1.” e) “+1,” “+0,” “−1.” f) “+0,” “+1,” “+0.” g) “+1,” “+0,” “+1.” The white dashed lines parallel to the “+0” operation direction (white arrow) are presented for the definition of the “0” destination as the center. The result of the addition operations can be obtained by the identification of the final destinations of the chiral stripe elongation, marked as purple at the right end and of (d–g).

defects during sputtering, where PMA energy is relatively low. Each three MOKE images in Figure 5d–g represent three successive additions from the left and at the right end, where the final destinations are classified by purple. As we designate the second operation as the “+0” operation (the white arrow), the destination point of the extended white arrow from the nucleation point (the white dashed line in the right images of Figure 5d–g) is referred as the “0” destination. Then, for the definition of the “+1” and “−1” operations, we defined the “−1,” “−2,” “+1,” “+2” destinations based on the “0” destination as the center. Then, we can achieve the successive additions just by identifying which destination points the chiral stripe domain elongation event reaches at after all three operations. For example, Figure 5d indicates the successive additions of “−1,” “+0,” “−1.” As we activated two “−1” operations, the final destination is two blocks away from the “0” destinations, which is the “−2” destination. As another example, Figure 5e represents the additions of “+1,” “+0,” “−1,” where the answer is zero. In Figure 5e, the final destination of the chiral stripe domain elongation event is the “0” destination, which corresponds to the answer, “0.” The formation of other half-skyrmions during operation was observed as in Figure 5e. This is because the domain wall is a kind of unstable states due to its relatively higher energy than the environment, which means easily changeable state. As the domain wall motion in a creep region is dominated by the stochastic behavior, the domain

walls also can elongate by forming another half-skyrmion as in Figure 5e.^[22] However, the elongation of another stripe domain at the side of the original stripe domain can never pass ahead the half-skyrmion of the original stripe domain because all the half-skyrmion has the same velocity regardless of the position. Therefore, as the additionally formed half-skyrmion can hardly interfere with operation, we believe that this effect can be avoided in the future application. Furthermore, Figure 5f,g show the additions of “+0,” “+1,” “+0” and “+1,” “+0,” “+1” along the same lines. Although we only present three types of operations and five destinations (output signal), the number of operations and destinations (output signal) can be determined by the duration of the writing pulse and the radius of the half-skyrmion. The deviated distance from the reference operation (e.g., “+0”) for the one step of operations is defined by the pulse durations and the size of each detection area is determined by the radius of the half-skyrmion. Furthermore, in the view of scaling, the skyrmion-based adder–subtractor present several advantages compared to the conventional CMOS circuit for addition/subtraction operations. First, the conventional CMOS circuit requires one half adder and one half subtractor, which contain 20 transistors at least.^[23] However, the skyrmion-based adder–subtractor allows us to do the same operations only by 3–4 transistors. In case of power consumption, the reported power consumption of basic half-adder is about several pJ.^[24] The power consumption of

half-skyrmion device can be estimated from the SOT-MRAM, because they share the core operation physics (SOT) and structures. According to the recent report,^[25] the SOT-MRAM also present several pJ. Therefore, the half-skyrmion adder/subtractor can keep its advantages in scaling with comparable power consumption with CMOS-based adder/subtractor. Second, the skyrmion-based adder–subtractor does not require any cash memory for the successive operations because of the accumulator ability, while the conventional CMOSs essentially require cash memories. Therefore, as the skyrmion is known to be stable even down to sub-10 nm scale, this proposed addition operation device suggests novel possibility for the next generation logic devices.

3. Conclusion

Our work demonstrated wide and precise control of the SkHE with respect to the injected current direction by deforming the internal structure of skyrmions. This approach is versatile and easy to integrate with a number of application devices. Although our work can only be applied to the half-skyrmion at this stage because we chose an external magnetic field as a tool to deform the internal structure, we believe that our result presents its own novelty by suggesting that the internal structure of skyrmions can serve as an efficient handle for the motion of magnetic skyrmions. Furthermore, based on the demonstrated precise control of 2D half-skyrmion dynamics, concise devices for the addition operation with the adder/subtractor were offered as a concrete example of 2D half-skyrmion-based spintronics applications. More broadly, by addressing full 2D dynamics of half-skyrmions, our work provides a comprehensive picture of the current-driven skyrmion motion beyond the conventional 1D-like unidimensional motion, whereby opening a new research subject for half-skyrmions electronics based on its own unique advantages. At this stage, the external magnetic field is used to control the half-skyrmion. Since the external magnetic field is not scalable, it can be difficult to develop into a real device. However, we expect that utilization of the field-like-SOT or magnetic elements can be an alternative to replace an external magnetic field for realization of nanometer-scale device application.^[26]

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

S.Y. and K.-W.M. contributed equally to this work. S.Y. and K.-W.M. designed the experiment. C.H. directed and supported this project. K.-W.M. and C.K. set up the measurement devices. S.Y., J.S., and J.H. fabricated the sample. S.Y. and K.-W.M. performed the experiments and analyzed the data. D.-H.K. and S.K.K. helped to describe the motion of half-skyrmions. S.Y., K.-W.M. and C.H. wrote the paper. All authors were involved in the discussion of the results and commented on the manuscript.

Keywords

adder, half-skyrmions, skyrmions, skyrmion Hall effect, subtractor

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- [1] a) S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, P. Böni, *Science* **2009**, 323, 915; b) X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, Y. Tokura, *Nature* **2010**, 465, 901.
- [2] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, A. Rosch, *Nat. Phys.* **2012**, 8, 301.
- [3] N. Nagaosa, Y. Tokura, *Nat. Nanotechnol.* **2013**, 8, 899.
- [4] a) L. Vistoli, W. Wang, A. Sander, Q. Zhu, B. Casals, R. Cichelero, A. Barthélémy, S. Fusil, G. Herranz, S. Valencia, R. Abrudan, E. Weschke, K. Nakazawa, H. Kohno, J. Santamaria, W. Wu, V. Garcia, M. Bibes, *Nat. Phys.* **2019**, 15, 67; b) A. Neubauer, C. Pfleiderer, B. Binz, A. Rosch, R. Ritz, P. G. Niklowitz, P. Böni, *Phys. Rev. Lett.* **2009**, 102, 186602.
- [5] W. Jiang, X. Zhang, G. Yu, W. Zhang, X. Wang, M. Benjamin Jungfleisch, John, E. Pearson, X. Cheng, O. Heinonen, K. L. Wang, Y. Zhou, A. Hoffmann, S. G. E. te Velthuis, *Nat. Phys.* **2017**, 13, 162.
- [6] K. Litzius, I. Lemes, B. Krüger, P. Bassirian, L. Caretta, K. Richter, F. Büttner, K. Sato, O. A. Tretiakov, J. Förster, R. M. Reeve, M. Weigand, I. Bykova, H. Stoll, G. Schütz, G. S. D. Beach, M. Kläui, *Nat. Phys.* **2017**, 13, 170.
- [7] a) F. Jonietz, S. Mühlbauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Böni, R. A. Duine, K. Everschor, M. Garst, A. Rosch, *Science* **2010**, 330, 1648; b) S. Woo, K. Litzius, B. Krüger, M.-Y. Im, L. Caretta, K. Richter, M. Mann, A. Krone, R. M. Reeve, M. Weigand, P. Agrawal, I. Lemes, M.-A. Mawass, P. Fischer, M. Kläui, G. S. D. Beach, *Nat. Mater.* **2016**, 15, 501; c) S. Woo, K. M. Song, X. Zhang, M. Ezawa, Y. Zhou, X. Liu, M. Weigand, S. Finizio, J. Raabe, M.-C. Park, K.-Y. Lee, J. W. Choi, B.-C. Min, H. C. Koo, J. Chang, *Nat. Electron.* **2018**, 1, 288; d) J. Sampaio, V. Cros, S. Rohart, A. Thiaville, A. Fert, *Nat. Nanotechnol.* **2013**, 8, 839.
- [8] R. Tomasello, E. Martinez, R. Zivieri, L. Torres, M. Carpentieri, G. Finocchio, *Sci. Rep.* **2015**, 4, 6784.
- [9] K. Wang, L. Qian, S.-C. Ying, G. Xiao, X. Wu, *Nanoscale* **2019**, 11, 6952.
- [10] a) X. Chen, W. Kang, D. Zhu, X. Zhang, N. Lei, Y. Zhang, Y. Zhou, W. Zhao, *Nanoscale* **2018**, 10, 6139; b) D. Prychynenko, M. Sitte, K. Litzius, B. Krüger, G. Bourianoff, M. Kläui, J. Sinova, K. Everschor-Sitte, *Phys. Rev. Appl.* **2018**, 9, 014034; c) S. Li, W. Kang, Y. Huang, X. Zhang, Y. Zhou, W. Zhao, *Nanotechnology* **2017**, 28, 31LT01.
- [11] a) S. Luo, M. Song, X. Li, Y. Zhang, J. Hong, X. Yang, X. Zou, N. Xu, L. You, *Nano Lett.* **2018**, 18, 1180; b) X. Zhang, M. Ezawa, Y. Zhou, *Sci. Rep.* **2015**, 5, 9400.
- [12] A. A. Thiele, *Phys. Rev. Lett.* **1973**, 30, 230.
- [13] X. Zhang, Y. Zhou, M. Ezawa, *Sci. Rep.* **2016**, 6, 24795.
- [14] a) X. Zhang, Y. Zhou, M. Ezawa, *Nat. Commun.* **2016**, 7, 10293; b) S. Woo, K. M. Song, X. Zhang, Y. Zhou, M. Ezawa, X. Liu, S. Finizio, J. Raabe, N. J. Lee, S.-I. Kim, S.-Y. Park, Y. Kim, J.-Y. Kim, D. Lee, O. Lee, J. W. Choi, B.-C. Min, H. C. Koo, J. Chang, *Nat. Commun.* **2018**, 9, 959.
- [15] a) Y. Hirata, D.-H. Kim, S. K. Kim, D.-K. Lee, S.-H. Oh, D.-Y. Kim, T. Nishimura, T. Okuno, Y. Futakawa, H. Yoshikawa, A. Tsukamoto, Y. Tserkovnyak, Y. Shiota, T. Moriyama, S.-B. Choe, K.-J. Lee, T. Ono, *Nat. Nanotechnol.* **2019**, 14, 232.

- [16] a) S. Zhang, X. Zhang, J. Zhang, A. Ganguly, J. Xia, Y. Wen, Q. Zhang, G. Yu, Z. Hou, W. Wang, Y. Peng, G. Xiao, A. Manchon, J. Kosel, Y. Zhou, X.-X. Zhang, *Sci. Adv.* **2020**, 6, eaay1876.
- [17] I. Limesh, F. Büttner, G. S. D. Beach, *Phys. Rev. B* **2017**, 95, 174423.
- [18] a) W. Jiang, P. Upadhyaya, W. Zhang, G. Yu, M. B. Jungfleisch, F. Y. Fradin, J. E. Pearson, Y. Tserkovnyak, K. L. Wang, O. Heinonen, S. G. E. te Velthuis, A. Hoffmann, *Science* **2015**, 349, 283; b) S.-Z. Lin, *Phys. Rev. B* **2016**, 94, 020402.
- [19] J. Iwasaki, M. Mochizuki, N. Nagaosa, *Nat. Commun.* **2013**, 4, 1463.
- [20] a) C. Reichhardt, C. J. O. Reichhardt, *New J. Phys.* **2016**, 18, 095005; b) W. Legrand, D. Maccariello, N. Reyren, K. Garcia, C. Moutafis, C. Moreau-Luchaire, S. Collin, K. Bouzehouane, V. Cros, A. Fert, *Nano Lett.* **2017**, 17, 2703; c) J.-V. Kim, M.-W. Yoo, *Appl. Phys. Lett.* **2017**, 110, 132404; d) R. Juge, S.-G. Je, D. d. S. Chaves, L. D. Buda-Prejbeanu, J. Peña-Garcia, J. Nath, I. M. Miron, K. G. Rana, L. Aballe, M. Foerster, F. Genuzio, T. O. Menteş, A. Locatelli, F. Maccherozzi, S. S. Dhesi, M. Belmeguenai, Y. Roussigné, S. Auffret, S. Pizzini, G. Gaudin, J. Vogel, O. Boulle, *Phys. Rev. Appl.* **2019**, 12, 044007.
- [21] F. Büttner, I. Limesh, G. S. D. Beach, *Sci. Rep.* **2018**, 8, 4464.
- [22] N. Saratz, T. Michlmayr, O. Portmann, U. Ramsperger, A. Vaterlaus, D. Pescia, *J. Phys. D: Appl. Phys.* **2007**, 40, 1268.
- [23] a) H. C. Lai, S. Muroga, *IEEE Trans. Comput.* **1979**, C-28, 648; b) T. Liu, K. R. Hohulin, L. E. Shiau, S. Muroga, *IEEE Trans. Comput.* **1974**, C-23, 63.
- [24] a) P. Kumar, N. S. Bhandari, L. Bhargav, R. Rathi, S. C. Yadav, presented at 2017 Int. Conf. Computing, Communication and Automation (ICCCA), Greater Noida, India, May 2017; b) A., N. Rajan, presented at 2019 Int. Conf. Intelligent Sustainable Systems (ICISS), Palladam, India, February 2019.
- [25] K. Garelo, F. Yasin, S. Couet, L. Souriau, J. Swerts, S. Rao, S. Van Beek, W. Kim, E. Liu, S. Kundu, D. Tsvetanova, K. Croes, N. Jossart, E. Grimaldi, M. Baumgartner, D. Crotti, A. Fumemont, P. Gambardella, presented at 2018 IEEE Symp. VLSI Circuits, Honolulu, HI, June 2018.
- [26] a) S. Joo, T. Kim, S. H. Shin, J. Y. Lim, J. Hong, J. D. Song, J. Chang, H.-W. Lee, K. Rhie, S. H. Han, K.-H. Shin, M. Johnson, *Nature* **2013**, 494, 72; b) R. Mishra, F. Mahfouzi, D. Kumar, K. Cai, M. Chen, X. Qiu, N. Kioussis, H. Yang, *Nat. Commun.* **2019**, 10, 248.